OPTIMISED PERMUTATION FILTER

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SUMMARY

This paper focuses the use of permutation theory in image processing, namely we present new results related to optimised permutation filters. Thus, to form an estimate in the environment corrupted by impulse noise, an optimised set of rank rules provides the best tool for outliers rejection.

Nonlinear filters based on permutation theory, called permutation filters, incorporate to a filter design both temporal-order and rank-order information included in a permutation group. Unlike a majority of rank-order based filters, where only rank-order information of an ordered input set is considered, by using the full potential of the permutation group transformation, an estimate accuracy given by optimised permutation filters is extremely high. In general, an optimisation algorithm does not exclude the use both mean absolute error and mean square error criteria. However, in the case of the mean absolute error criteria, the optimised permutation filter should provide the signal-details preservation rather than the noise attenuation and contrariwise the achieving of the optimised permutation filter under minimisation the mean square error criteria is characterised by the noise suppression rather than the preservation of image features.

Based on above discussion, the novelty of this paper lies at the analysis the optimisation parameters such as "forgetting factor" and optimisation criteria. We will show that in the case of small window sizes (e.g. a window size equal to five), there is necessary to achieve the best balance between the noise suppression and the signal-details preservation. Thus, we provide an optimisation criteria compromise. Next, the influence of used optimisation criteria is decreased with an increased window size, since a permutation group size is larger considerably than an input sequence. For this "static" case, we provide the dependence of estimation accuracy on the "forgetting" parameter. All above conclusions are supported by a number of figures and tables.

Keywords: *noisy image filtering, impulse noise, permutation filter, rank information, temporal information.*

1. INDRODUCTION

The employment of nonlinear filters [1,2,5,9,11] can markedly improve the estimation efficiency in a wide range of smoothing applications, where the environment is modelled as nonlinear or when the noise corruption is nonGaussian. The typical example of the situation related to both nonlinear environment and nonGaussian noisy process, simultaneously, is represented by useful image information corrupted by the impulse noise or outliers. When something is nonlinear, it means that the superposition property cannot be applied and thus, the use of nonlinear filters on uncorrupted signal (useful information) and the noise, each separately, is impossible.

To suppress the impulse noise, usually the robust order-statistics theory [6,8,13] is used. The nonlinearity of order-statistic filters [3,5,7,12] is caused by an ordering of input set. A class of order-statistic filters avoids the failures of linear filters that tend to introduce a blurring of signal-details. In addition, linear filters produce new samples in a resulting image. In the case of order-statistic filters, a filter output is a sample from the input set.

The performance of chosen filtering method depends especially on the measure of the accuracy of the solution. Of course, there is necessary to achieve a compromise between the noise suppression and the signal-details preservation. Since filter nonlinearity, the optimal filtering situation, when only affected samples are processed, cannot be fully obtained. For that reason, the searching of efficient estimation algorithms is still actual. One way how to come near to the optimal situation with simultaneous use the robust order-statistics theory lies at the consideration both rank-order and temporal order information contained in the input set. A class of order-statistic permutation filters [2,3], utilises the full potential of the permutation group and thus, it gives the assumption for successful use in smoothing applications.

This paper focuses the analysis of optimisation parameters in the acquisition process of optimised permutation filters [1]. Following analysis provide simplification for additional use of optimal set of rank rules and the future implementation of permutation filters, too.

2. PERMUTATION FILTERS

Unlike some order-statistic filters, namely rank-order filters, a class of permutation filters utilises both rank order and temporal order information included in the input set $\mathbf{x}$. Rank-order filters [8,11,13] are based on the relationship between input vector $\mathbf{x}$ and its rank-ordered version $\mathbf{x}^r$ and thus, these filters ignore temporal information contained in $\mathbf{x}$. The temporal information lies at changes of sample positions in $\mathbf{x}$. By this way, there is possible to generate $N!$ different permutations of $\mathbf{x}$, however, all permutations result to the same rank-ordered vector $\mathbf{x}^r$. For that reason, the output of permutation filters is a function of the permutation that maps $\mathbf{x}$ to $\mathbf{x}^r$ [2,3,4].

Let n be a time position of input sequence and

$$\mathbf{x}(n) = [x(n-K),...,x(n),...,x(n+K)] \quad (1)$$

$$= [x_1(n), x_2(n),..., x_N(n)] \quad (2)$$

an input set in this time position. Note that the input set $\mathbf{x}(n) \in R^N$ is determined by a symmetrical window shape with a size $N = 2K+1$. The time position of the window shape is given by central sample $x(n) = x_{(N+1)/2}(n)$. The ordering of the input set $\mathbf{x}(n)$ means that samples $x_1(n), x_2(n),..., x_N(n)$ are mapped to the rank-ordered vector [13]

$$\mathbf{x}^r(n) = [x_{(1)}(n), x_{(2)}(n),..., x_{(N)}(n)] \quad (3)$$

where $x_{(1)}(n), x_{(2)}(n),..., x_{(N)}(n)$ defined by

$$x_{(1)}(n) \le x_{(2)}(n) \le ... \le x_{(N)}(n) \quad (4)$$

represent order-statistics.

For better understanding of permutation filtering operation, we consider following simplicity. Let $\mathbf{x} = [x_1, x_2,..., x_N]$ be input set or temporal-ordered vector and $\mathbf{x}^r = [x_{(1)}, x_{(2)},..., x_{(N)}]$ rank-ordered vector. Then the observation permutation $p_\mathbf{x}$ is defined as the permutation that maps $\mathbf{x}$ to $\mathbf{x}^r$. Above operation can be expressed as [2,4]

$$\mathbf{x}p_\mathbf{x} = [x_{1p_\mathbf{x}^{-1}}, x_{2p_\mathbf{x}^{-1}},..., x_{Np_\mathbf{x}^{-1}}] \quad (5)$$

$$= [x_{(1)}, x_{(2)},..., x_{(N)}] \quad (6)$$

$$= \mathbf{x}^r \quad (7)$$

where the mapping from the temporal-ordered indices to rank-ordered indices determines the output rank decision $l_\mathbf{x}$. It is clear that the set of temporal-ordered indices $\Omega_N = [1,2,...,N]$ of the input set $\mathbf{x}$ is mapped to the same set Ω_N which is characterised by the change of element position, only, caused by the sample ordering in $\mathbf{x}^r$. Note that all permutations of Ω_N form the group of permutations [2,3,10].

Since the output of permutation filters is always the order-statistic from $\{x_{(1)}, x_{(2)},..., x_{(N)}\}$, then it is possible to separate the group of permutations to N subsets H_i, for $i = 1,2,...,N$, called blocks of partition. It is clear that each H_i is associated with the order-statistic $x_{(i)}$. Since $H_1, H_2,..., H_N$ have to be pairwise disjoint and their union is the group of permutations, the set $\mathbf{H} = \{H_1, H_2,..., H_N\}$ is simply called partition on the group of permutations. In the case of sample equivalence, some blocks H_i can be empty subsets. The set of all partitions is described as $\Omega_\mathbf{H}$.

The output of permutation filter is defined by

$$F_P(\mathbf{x};\mathbf{H}) = x_{(l_\mathbf{x})} \quad (8)$$

where $\mathbf{x}$ is input set and $\mathbf{H} \in \Omega_\mathbf{H}$ is a partition. Block associated with the $l_\mathbf{x}$th order-statistic contains the observation permutation.

To come near to the optimal filtering situation, there is necessary to achieve the optimal set of $N!$ (for each possible permutation) output rank rules $l_\mathbf{x}$.

3. OPTIMISATION

The aim of the optimisation is to set the decision vector $l = (l_1, l_2,..., l_{N!})$, where

$$p_i \in \mathbf{H}_{l_i} \text{ for } i = 1,2,...,N! \quad (9)$$

The optimal filter can be obtained by optimising each of the elements l_i in vector l independently.

Let $\{o\}$ be original and $\{x\}$ noisy training sequence, both of Q elements. In general, the sum of L_γ normed errors to be minimised can be expressed as [2,3]

$$\sum_{n=1}^{Q} \lambda^{(Q-n)} | o(n) - F_P(\mathbf{x};\mathbf{H}) |^\gamma \quad (10)$$

where $\lambda \in (0,1]$ is "forgetting" factor, and $F_P(\mathbf{x};\mathbf{H})$ is the output of permutation filter. If $\gamma = 1$, then the optimised filter will be characterised by preserving characteristics, while $\gamma = 2$ determines that the permutation filter provides the noise attenuation characteristics. However, as will be shown in the next section, the influence of used metrics γ to a filter behaviour decreases with an increasing window size.

To provide better understanding of above optimisation, we simplify above optimisation in dependence on time n. Let $\mathbf{P}(n) = [P_1(n), P_2(n),..., P_N(n)]$ be a vector of L_γ normed errors at time n, where each [3,4]

$$P_i(n) = | o(n) - x_{(i)}(n) |^\gamma \text{ for } i = 1,2,...,N \quad (11)$$

characterises the error between the desired and the ith order statistic. Note that $x_{(i)}$ is one of N possible outputs.

Mark $\mathbf{R}_i(n) = [R_{i,1}(n), R_{i,2}(n),..., R_{i,N}(n)]$ as a vector containing estimation (cumulative) errors so that i characterises observation with permutation index. The accordance between j and permutation index i determines the update of each $\mathbf{R}_j(n)$, for $j = 1,2,...,N!$, with the consideration $\mathbf{P}(n)$, i.e. vector of L_γ normed errors. Mathematically, above operation can be described by

$$\mathbf{R}_j(n) = \begin{cases} \lambda \mathbf{R}_j(n-1) + \mathbf{P}(n) & \text{if } i = j \\ \lambda \mathbf{R}_j(n-1) & \text{otherwise} \end{cases} \quad (12)$$

The optimal l_i term is found by simply searching $\mathbf{R}_i(n)$ for the minimum cumulative error, i.e.

$$l_i = k \text{ for } R_{i,k}(n) = \min\{R_{i,1}(n), R_{i,2}(n),..., R_{i,N}(n)\} \quad (13)$$

The initialisation includes following operations such as the set-up $\mathbf{R}_i(0) = 0$ and $l_i = (N+1)/2$, both for $i = 1,2,...,N!$. Note that sequence position starts with $n = 1$. Although initial setting of each l_i is arbitrary from the set $\{1,2,...,N\}$, there can occur

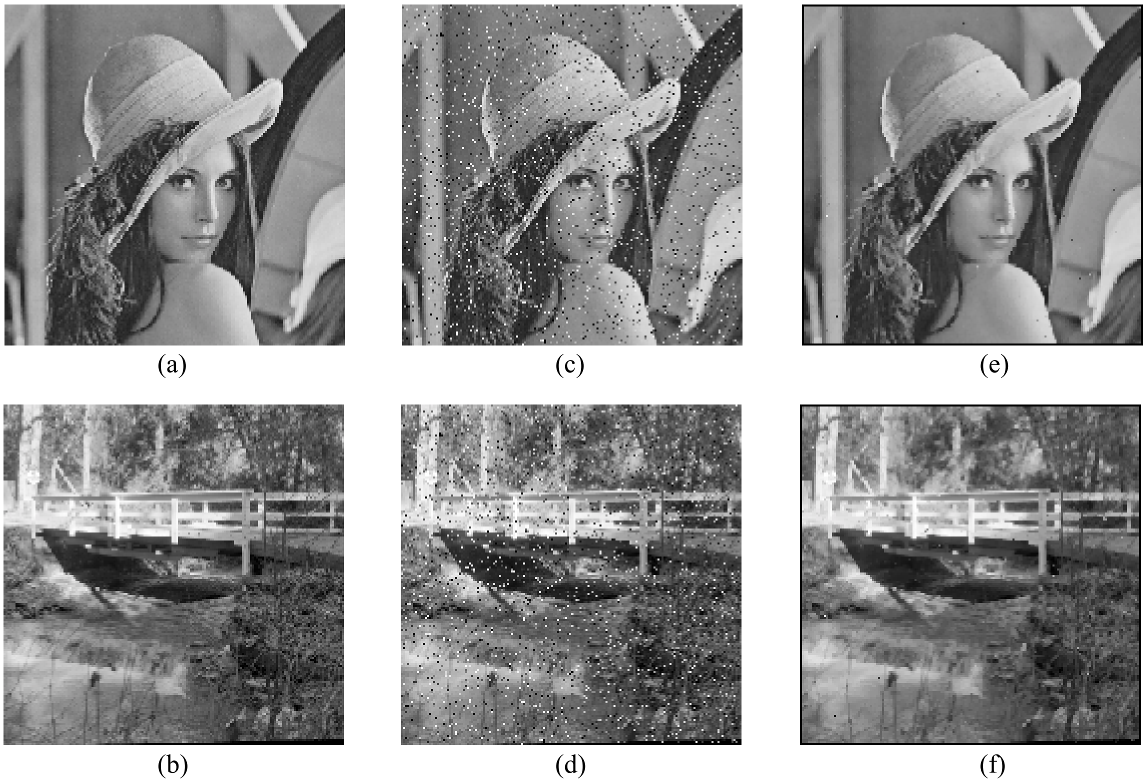

Fig. 1 Achieved results
(a) original image Lena (b) original image Bridge (c) 10% random-valued noise
(d) 10% random-valued impulse noise (e) median with +3 window shape (+3) (f) median (+3)

situations, where some l_i is never updated. For that reason, it is appropriate to set each l_i to the robust output rank, namely median rank $(N+1)/2$.

The optimisation consists of the repeating steps (9), (11), (12) and (13) and it is finished at the end of training sequences, i.e. for $n = Q$, or when filter is sufficiently trained.

4. SIMULATION RESULTS

As the test images, two gray-scale images of different statistical properties were used (Fig.1a,b). Each image has a resolution of 256×256 pixels with 8-bits/pixel gray-scale quantization. The complexity of an image is evaluated with regards to the problem areas such as image details and edges.

The first image, well-known Lena (Fig.1a), includes a number of details and large monotonous field, too. From the filtering aspect, the problem areas are represented by women hair and eyes. Image Bridge (Fig.1b) is more complex, there are many edges. Small objects such as vegetation, trees and bush will represent places, where the filter behaviour can be very problematic.

To illustrate the degree of damage, we use the model of random-valued impulse noise. This noise type (Fig.1c,d) replaces some of the image pixels by gray pixels, in the case of 8 bit-quantized image by the value from 0 to 255. The mathematical formula for variable valued impulse noise is given by [8]

$$x(n) = \begin{cases} \upsilon & \text{with probability } p_\upsilon \\ o(n) & \text{with probability } 1\text{-}p_\upsilon \end{cases} \qquad (14)$$

where $x(n)$ is noisy image signal, $o(n)$ describes original image signal, both in the time position n and υ represent an random value, i.e. impulse, with the occurrence probability p_υ.

The difference from original was evaluated by two criteria, namely the mean absolute error (MAE) and mean square error (MSE). The first criteria evaluates the signal-details preservation well, whereas the second one is a mirror of the noise suppression. Two-dimensional definitions of MAE and MSE are expressed as [8]

$$MAE = \frac{1}{AB}\sum_{i=1}^{A}\sum_{j=1}^{B}\left|o_{i,j} - x_{i,j}\right| \qquad (15)$$

$$MSE = \frac{1}{AB}\sum_{i=1}^{A}\sum_{j=1}^{B}\left(o_{i,j} - x_{i,j}\right)^2 \qquad (16)$$

where A, B represent image dimensions and i, j determine time position, i.e. $n = iB + j$.

Now, we present some simulation results. At first, we observed the filter behaviour in dependence on L_γ normed error (Fig.3). In the case of +3 window shape (it is cross window of five samples), the best balance between the noise suppression and the signal-details preservation was achieved for $\gamma = 1.5$. For square window 3x3, i.e. nine samples, the filter behaviour is relatively constant. It means, that in this case the used norm can obtain the arbitrary value.

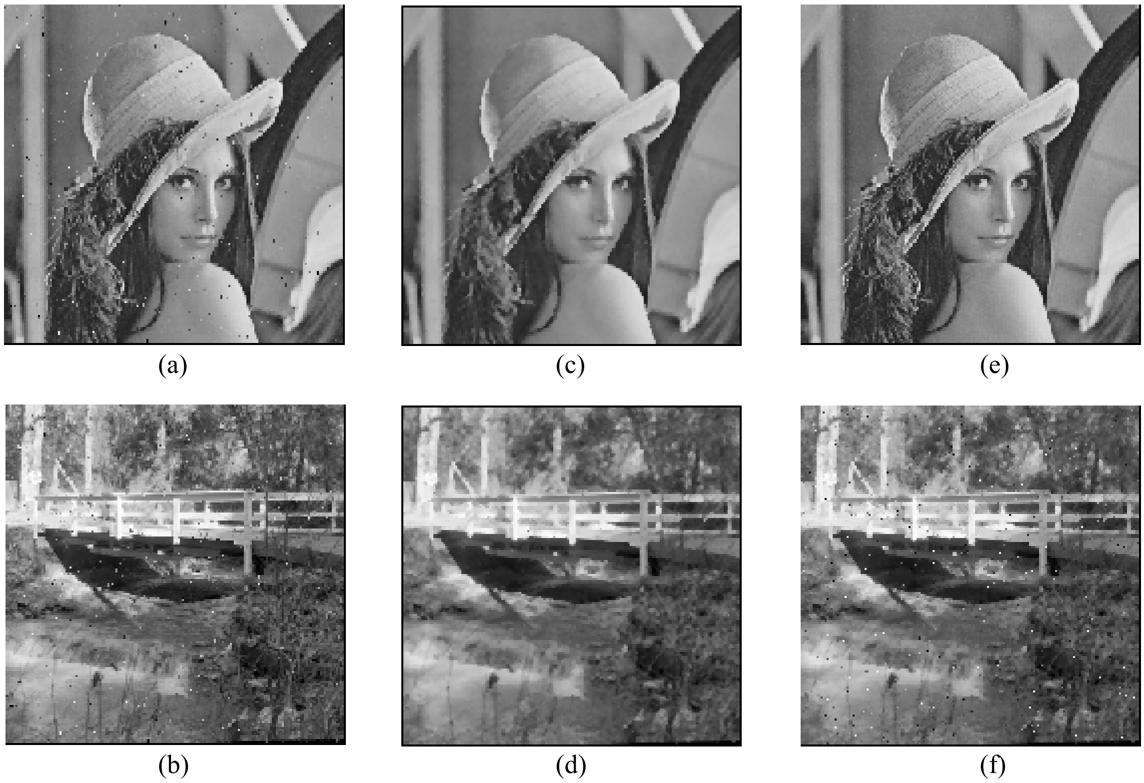

Fig. 2 Achieved results
(a) permutation filter (+3, $\lambda = 1$, $\gamma = 1$) (b) permutation filter (+3, $\lambda = 1$, $\gamma = 1$) (c) median (3x3)
(d) median (3x3) (e) permutation filter (3x3, $\lambda = 1$, $\gamma = 1$) (f) permutation filter (3x3, $\lambda = 1$, $\gamma = 1$)

Optimisation	**Optimised on Lena**				**Optimised on Bridge**			
Image	***Lena***		***Bridge***		***Lena***		***Bridge***	
Method	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**
Identity	7.018	759.1	7.221	807.6	7.018	759.1	7.221	807.6
Median (+3)	4.109	82.0	5.935	**123.7**	4.109	**82.0**	5.935	123.7
Median (3x3)	4.888	94.3	8.042	173.7	4.888	94.3	8.042	173.7
Permutation filter (+3)	2.262	102.1	**3.582**	138.9	**2.784**	121.2	3.395	144.6
Permutation filter (3x3)	**0.301**	**3.0**	7.595	232.2	4.795	164.8	**0.479**	**6.1**

Table 1 Performance of proposed methods (permutation filter: $\lambda = 1$, $\gamma = 1$)

To obtain the objective comparison of the filter performance (Fig.2a,b and Fig.2e,f), achieved results were compared with a median filter (Fig.1e,f and Fig.2c,d), i.e. with the initial set of output rank rules.

The optimised permutation filter (Table 1) smoothes (Fig.2e) the noise excellent and preserves signal-details, simultaneously. However, the optimised permutation filter provides above excellent behaviour for the training sequence, only. Sequences with different statistical properties (Fig.2f) are processed insufficiently. It means that optimised set of output rank rules is not robust.

When our attention is focused on the influence of parameter λ (Fig.4), the best filter behaviour is observed for the maximal possible value, i.e. $\lambda = 1$. Exponential character of λ exercises especially at the beginning of the training sequence, since $\lambda < 1$ involved by a high number results to close zero.

5. CONCLUSION

In this paper, new results and parametric analysis of the optimisation for permutation filters were presented. Simulation results showed us that for a window size $N = 9$, the choice of minimisation criteria (MAE, MSE) is not important. In the case of "forgetting" factor λ, the best results were achieved for $\lambda = 1$. The only problem is to optimise the set of output ranks so that the future research should be oriented to the searching for faster optimisation.

Considering worse results not achieved on training sequences, the additional research task is related to robustness improving of optimised permutation filters so that these filters should represent the most efficient filtering algorithm for nonGaussian noisy processes. For that reason, the key role will be played by simplification of the filter implementation.

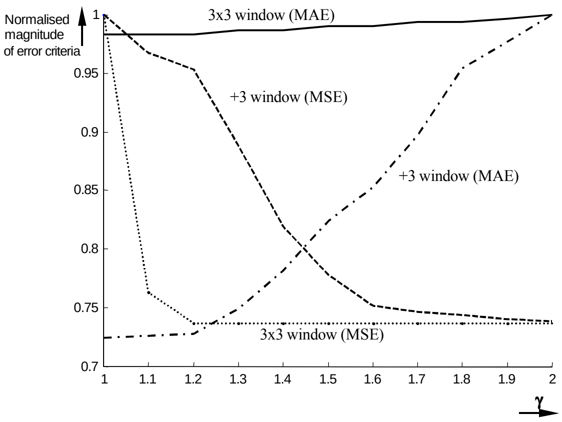

Fig. 3 Dependence of normalised error criteria (MAE, MSE) on parameter γ

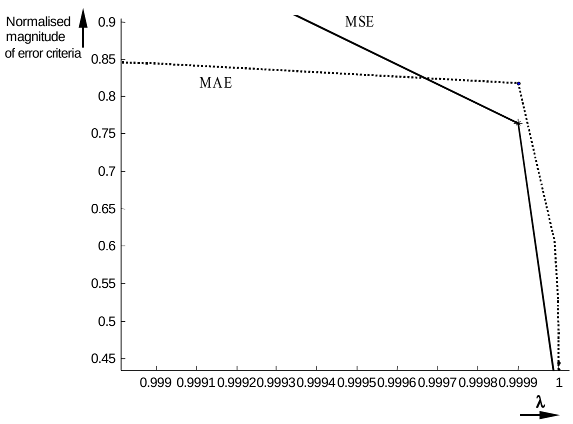

Fig. 4 Dependence of normalised error criteria (MAE, MSE) on parameter λ

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BIOGRAPHY

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