MODEL AND CONTROL OF A SWITCHED RELUCTANCE MOTOR

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SUMMARY

A switched reluctance motor is described by partial differential equations and the corresponding mathematical model is relatively complicated. Many methods trying to simplify it were published during last decades. This paper deals with one possible scheme convenient for bi-directional motor speed control. Non-linear functions of phase current and torque were modelled by means of look-up tables containing measured values. Mutual position of stator and rotor poles of a motor with pole ratio 3:2 repeats after each ninety degrees, therefore rotor angle and non-linear functions are calculated for this interval only. Logic unit enables timing of voltage pulses due to the required direction of rotation and braking. Simulation scheme was realised by means of the MATLAB programme. Time responses of rotor angle, speed, phase currents and torque were drawn at simulation and they are well matching with those obtained by measurement.

Keywords: *switched reluctance motor, speed control, simulation.*

1. INTRODUCTION

Though the switched reluctance motor (SRM) was designed in 1842 its research and development has begun after 1980, when results of P.Lawrenson and his team were published [5]. Many papers offer mathematical models and methods suitable for motor control. This article is based on measured phase inductance *L(i,* Θ) of a motor with pole ratio 3:2, which is depicted in Fig.1.

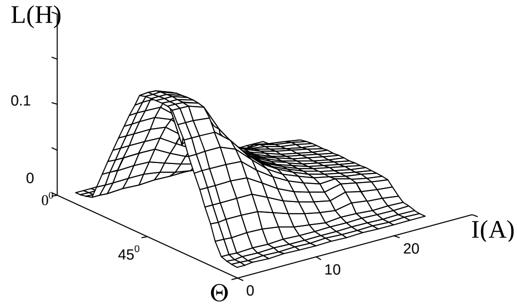

Fig.1. Inductance *L(i,* Θ) as function of phase current and rotor angle

Phase current and torque are described by following equations

$$U = R\,i + \frac{d\Psi(i,\Theta)}{dt} = R\,i + \frac{\partial\,\Psi(i,\Theta)}{\partial\, i}\frac{di}{dt} + \frac{\partial\,\Psi(i,\Theta)}{\partial\,\Theta}\frac{d\Theta}{dt}, \qquad (1)$$

$$M(i,\Theta) = \frac{d}{d\Theta}\left(\int_0^i \Psi(i,\Theta)di\right). \qquad (2)$$

Consideration of these non-linear equations is the main problem at design of a SRM mathematical model and control. More approaches have been derived to solve this task:

- SPICE programme enables to determine values of inductance corresponding to the constant speed in each integration cycle. Electrical loop is solved but not the corresponding torque.
- The real function of $\Psi(i,\Theta)$ is substituted by more convenient, usually exponential function, which enables integration. The following substitution is known [9]:
$$\Psi(i,\Theta) = a_1(\Theta)\left[1 - e^{a_2(\Theta)i}\right] + a_3(\Theta), \qquad (3)$$
where $a_i(\Theta)$ are functions expressed by Fourier series.
- Another substitution uses the following function:
$$\Psi(i,\Theta) = \Psi_{sat}\left(1 - e^{-i\,f(\Theta)}\right), \text{ where}$$
$$f(\Theta) = a + \sum_{n=1}^{\infty} b_n \sin(k_1\Theta - k_2). \qquad (4)$$
This function enables simple derivation and integration. Control of SRM by this solution was successfully verified by simulation [3].
- A convenient substitution function may be obtained from measured or calculated values by a bi-cubic spline interpolation, because it gives smooth curves and enables to realise on-line control [7], [8].
- Very precise solution is reachable by finite element method [2]. The input magnitudes are stator and rotor dimensions, number of windings and saturation curve. Division of magnetic part in more thousand elements ensures exactness. A powerful computer, corresponding programme and higher computing time are needed.
- Reference [1] contains a numerical method applying look-up table of measured or computed values of $\Psi(i,\Theta)$. Then the inverse table i(ψ,Θ) is arranged and after numerical integration the

torque table $M(i,\Theta)$ is obtained. Instantaneous values of current *i* and torque *M* are calculated by interpolation.

- A special observer may determine motor average torque according to the following expression [4]:

$$M = \frac{N_m p}{2\phi\pi} \oint \int_0^j \frac{\partial \Psi(i,\Theta)}{\partial \Theta} di\, d\Theta \,, \tag{5}$$

where N_m and p are the number of phases and poles, respectively.The magnetic flux of a phase may be determined as $\Psi = \int (u - Ri) dt$. This control method has found application in electrical traction.

2. SRM MODEL

The mathematical model of the SRM drive including its control is shown in Fig.8. Particular subsystems are described below.

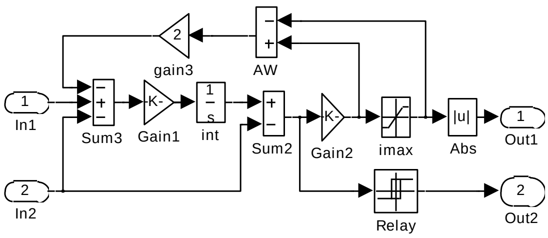

Fig.2. Speed controller

Compensation of the load torque requires an integral in the speed controller (Fig.2). Anti wind up scheme is added for to avoid the output value overshoot caused by the presence of a limiter.

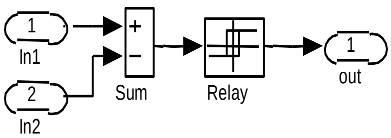

Fig.3. Model of a pulse converter

Current deviation is connected to the input of a relay element, which corresponds to a pulse-modulated converter (Fig.3). Its output voltage varies within the interval <+U, -U >.

Voltage pulse distribution to the corresponding phase winding is realised by a logic unit. (Fig.4). Because mutual position of stator and rotor poles repeats after 90°, it is satisfactory to connect each phase ones during this interval for approximately 30°. If driving torque is required, then phase winding should be fed at raising inductance function. A braking torque is produced at negative derivation of inductance function. Switching of both control modes may be derived from the speed deviation sign (input delta). Phase shifting by plus 60° or minus 30° necessary for the required direction of rotation or braking may be realised by changing phase outputs of the logic unit (switches B1, B2 and B3). Fast decreasing of winding currents is ensured by negative voltage outside of voltage pulse.

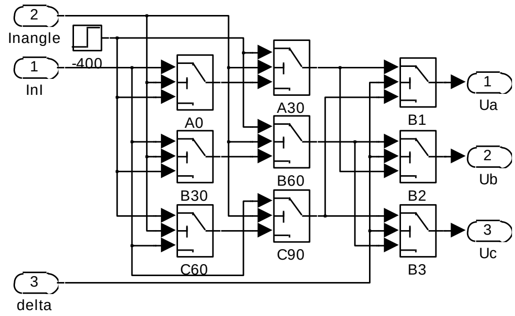

Fig.4. Model of the logic unit

Current loop contains non-linear function *L(i,*Θ). The winding model of one phase (Fig.5) is given by the following non-linear equation:

$$i = \int \frac{1}{L(\theta,i)} (u - Ri - \frac{\partial L(\theta,i)}{\partial t} i) dt \tag{6}$$

Because current can reach only positive values the lower integral limit is equal zero. The inductance functions for particular phases were embedded into look-up tables. Current feedback to the converter is calculated as sum of all phase currents.

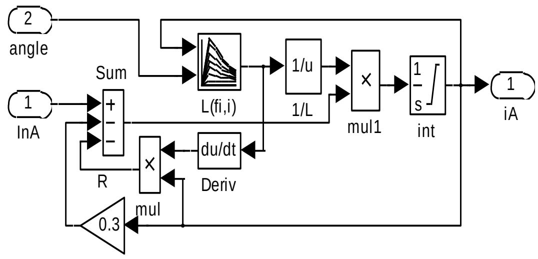

Fig.5. Model of one phase winding.

Phase torque of each phase as function of current and rotor position (angle) may be also expressed by means of look-up tables. Since phase currents are always positive, torque sign is determined by mutual poles position, i.e. by due timing of the logic unit. Model of this subsystem is shown in Fig.6, output of which yields the sum of phase torques. Load torque is considered as passive and constant for both rotation directions.

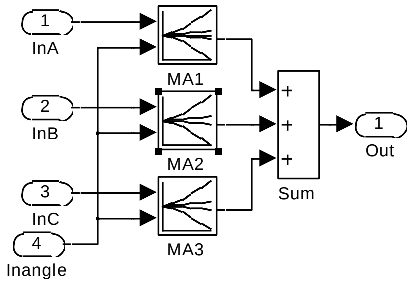

Fig.6. Model of the torque creating subsystem

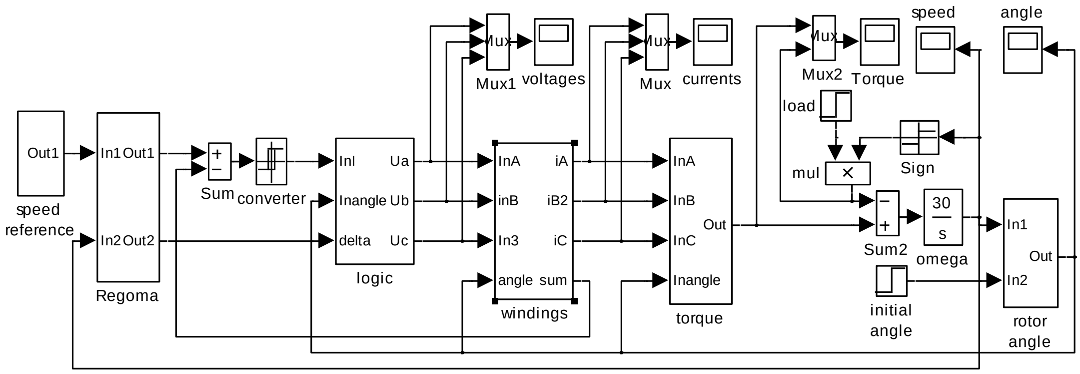

Fig.8. Simulation scheme of the SRM

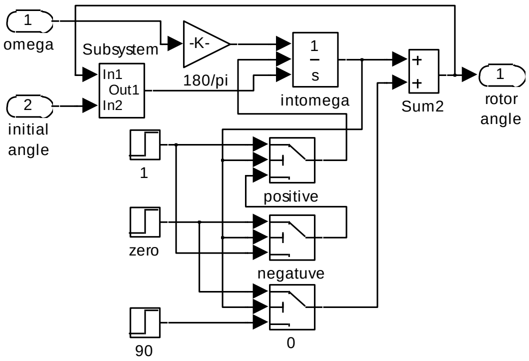

Fig.7. Rotor angle calculation

Motor model also contains determination of the instantaneous rotor angle from the known angular speed (Fig.7). Distribution of converter impulses is repeating after each 90^{o} for a motor with pole ratio 3:2, therefore model is designed for this interval only. It should be also distinguished direction of rotation because forms of time responses are different as can be seen in Fig. 9.

3.SIMULATION RESULTS

SRM model was programmed in MATLAB (Fig.8) Time responses obtained at simulation are shown in Fig.9. Minimal inductance and torque characterise zero position, therefore start from this operating point should be realised at no-load. Since the first part of the speed curve is slow, time responses were drawn at 5^{o} of the initial position. According to the speed reference SRM is running up in one direction, then is reversed and loaded at time 0.43 s. Speed controller's output is limited to determine the highest phase current. Its second output yields signum value necessary for switching-over the logic unit due to the required direction of rotation and braking. Because motor torque is formed from pulses the obtained speed response cannot be smooth unless the moment of inertia is satisfactorily high.

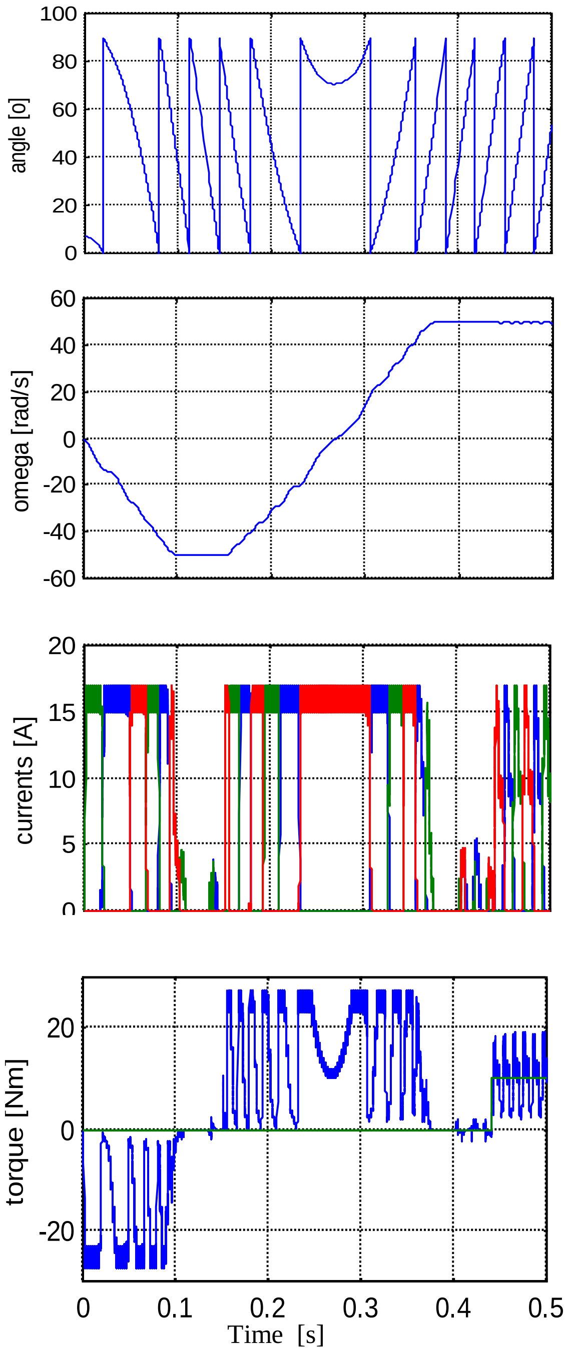

Fig.9. Simulation results

The presented SRM model may be verified by comparison of current oscillograms obtained at measurement on a real motor with current curves drawn at simulation.

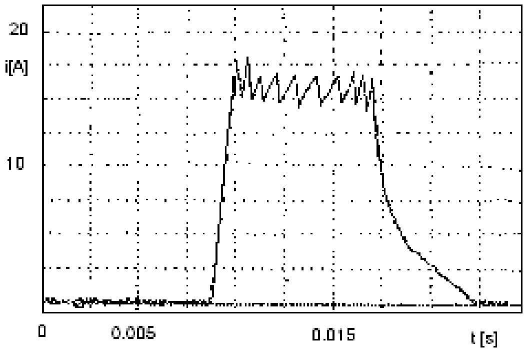

Fig.10 Phase current form at 60 rad/s.

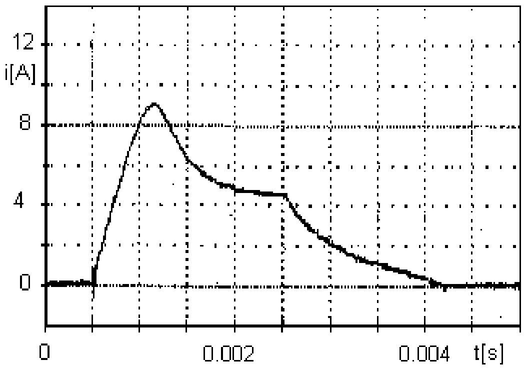

Fig.11 Phase current form at 233 rad/s.

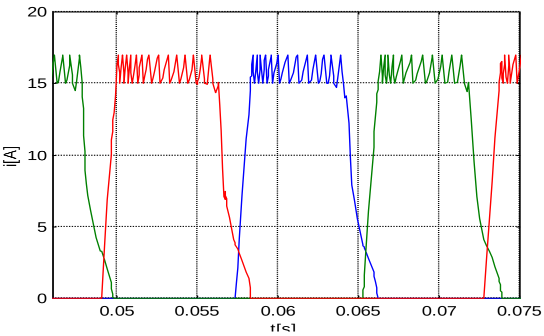

Fig.12 Simulated currents at 60 rad/s.

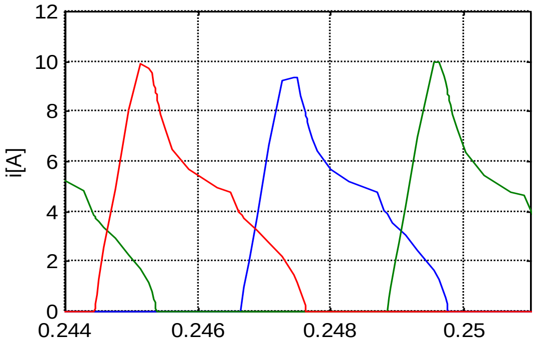

Fig.13 Simulated currents at 233 rad/s.

Detailed forms of currents are shown in Fig.10 and Fig.11. They were drawn at different angular speed and at 230 V of voltage pulse. Rated power of the measured SRM was 4 kW at 3000 rev./min. Further parameters were published in [11]. The following Fig.12 and Fig.13 contain corresponding time responses of all three phases obtained at simulation

4. CONCLUSION

The above introduced relatively simple simulation scheme enables to illustrate basic properties of a SRM. Instead of approximate mathematical expressions here was preferred measuring of non-linear functions and their embedding in D2 look-up tables. This model enables study the drive behaviour at chosen initial rotor position, setting the load torque at given operating point, pulse voltage variation as function of speed, easy changing of turn-on and turn-off angles and further SRM properties. The simulation results are satisfactorily matching with measurement.

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BIOGRAPHY

Želmíra Ferková (Ing. PhD) graduated in electrical engineering from the Technical University Košice in 1982 and received her PhD in 1994. At present she is a senior lecturer of electric machines at the Department of Electrical Drives and Mechatronics TU Košice. Her professional area is design of electrical machines , mainly of switched reluctance motor and its performance.

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