SENSITIVITY ANALYSIS OF SPEED AND POSITION OBSERVERS FOR THE PERMANENT MAGNET SYNCHRONOUS MOTOR

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SUMMARY

In the paper three methods for speed and position estimation for the permanent magnet synchronous motor were described and compared. The first method is based on voltage differential equations of PMSM and the second one uses the reduced-order observer of Luenberger type. The third method uses the full-order extended observer. The rotor speed and position are estimated based on the line current measurements for the motor. The mathematical model of the vector-controlled drive system in Matlab-Simulink was used for simulation. All the estimation methods were compared and evaluated from the point of view of model sensitivity to the stator parameter changes.

Keywords: *Permanent magnet synchronous motor, sensorless control, speed estimation, state observer.*

1. INTRODUCTION

For some years a wide interest in the development of more reliable drive systems is observed. This is associated with the elimination of mechanical sensors, which generate additional costs and cabling, which is specially important in the machine tools and robotic applications. So microprocessors systems are developed, which estimate the signals necessary for the realisation of feedback loops. In the drive systems with PMSM a tendency to the elimination of position sensors is also observed. Drive systems with PMSM offer better properties than systems with the induction motor, especially in case of systems with high dynamical requirements, for example in servodrives [12]. But these properties are dependent on suitable commutation of converter valves what requires the information about actual rotor position. Thus the estimation problem of this state variable becomes particularly important.

In the recent years a few methods of speed and position reconstruction of permanent magnet synchronous motor were proposed:

- the method based on the stator flux vector calculation [2], using measurements of stator line currents and voltages;
- the method based on calculation of the back electromotive force of the rotor using motor voltage equation [6], [11], [14];
- the method of correction of the hypothetical rotor position [8], based on the calculation of the actual rotor position using a difference between measured voltage and voltage calculated from the stator equation;
- the method based on the full-order [3] or the reduced-order [13] state observers;
- the method based on the Kalman filter [1];
- the method using information about changes of the voltage inducted in the stator winding as the result of a difference between reluctance in x and y axes [5];
- the method based on a sliding mode observer [15].

The paper shows results of analysis of three above-mentioned methods of the reconstruction of PMSM speed and position:

- the method based on calculation of the back electromotive force of the rotor using motor voltage equation [6], [11], [14];
- the method based on a reduced-order state observer of back electromotive force, modified in the comparison with method described in [13];
- the method based on an extended full-order state observer, modified in the comparison with [3].

The analysis of these methods was performed from the point of view of their sensitivity to motor parameter changes and was based on the simulation of the drive system.

2. MATHEMATICAL MODEL OF THE DRIVE SYSTEM

2.1. Motor equations

In the simulation the mathematical model of the permanent magnets synchronous motor was used with usual assumptions: constant parameters of the motor equivalent circuit, nonlinear effects neglected, sinusoidal back electromagnetic force, symmetry of the motor, temperature influence to motor parameters as well as permanent magnets neglected. After the transformation of the three-phase equations of the motor to rectangular co-ordinate system [α,β], this mathematical model is described as follows:

- stator equations:

$$U_{s\alpha} = R_s \cdot i_{s\alpha} + L_s \frac{di_{s\alpha}}{dt} + k_e \cdot \omega \cdot \sin(\Theta)$$
$$U_{s\beta} = R_s \cdot i_{s\beta} + L_s \frac{di_{s\beta}}{dt} + k_e \cdot \omega \cdot \cos(\Theta) \tag{1}$$

where k_e is voltage constant of the motor [Vs/rad];

- equation of the electromagnetic torque T_e:

$$T_e = k_e \cdot (i_{s\alpha} \cdot \sin\Theta + i_{s\beta} \cdot \cos\Theta) \tag{2}$$

-motion equation:

$$\frac{d\omega}{dt} = \frac{1}{J} \cdot (T_e - T_L) \tag{3}$$

$$\omega = \frac{d\Theta}{dt} \tag{4}$$

2.2.The structure of the drive system

The vector-controlled system of PMSM drive was taken into account. To assure the effect of "smooth" electromagnetic torque of the motor, the exact information about the rotor angular position is necessary. The schematic diagram of the control system with the direct orientation of the current vector regarding the rotor flux vector (based on calculated components of the stator current vector in [d,q] axes) is presented in Fig.1. Output signal of speed controller determines the input value of slave torque controller – which in fact is the controller of i_{sq} current component. Because the horizontal component of the stator current vector $i_{sd} = 0$, this i_{sq} current controller simultaneously influence the absolute value of the stator current vector. The PI controllers in this structure were optimised by using the modulus and symmetry criteria [4].

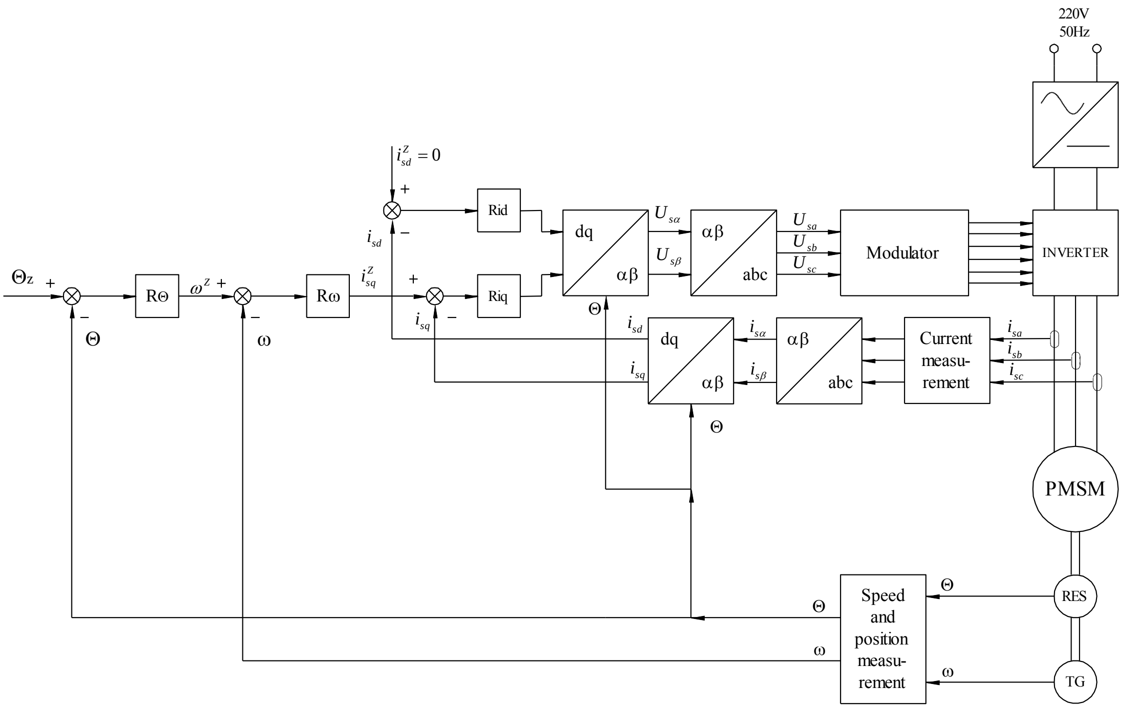

Fig.1 Schematic diagram of the vector-controlled drive system with permanent magnet synchronous motor

3. SYSTEMS FOR SPEED AND POSITION ESTIMATION

3.1. Speed and position estimator based on the voltage equation of the motor

Taking into account the voltage equation (1) in [α,β] axes, the simple transformation to the model, which reconstructs the sine and cosine functions of the rotor position is possible, based on the easily measured quantities only – the line currents and voltages of the stator winding [11].

Thus the estimator of the rotor position angle is obtained in the following way:

$$\cos\hat{\Theta} = \frac{L_s}{k_e} \cdot i_{s\alpha} - \frac{1}{k_e} \int_t (U_{s\alpha} - R_s \cdot i_{s\alpha}) dt$$
$$\sin\hat{\Theta} = \frac{L_s}{k_e} \cdot i_{s\beta} - \frac{1}{k_e} \int_t (U_{s\beta} - R_s \cdot i_{s\beta}) dt \tag{5}$$

$$\hat{\Theta} = \arctan\left(\frac{\sin\hat{\Theta}}{\cos\hat{\Theta}}\right) \tag{6}$$

$$\hat{\omega} = \frac{d\hat{\Theta}}{dt} \tag{7}$$

3.2.The reduced-order state observer

The mathematical model (1) of the PMSM can be described in the matrix form as following:

$$\begin{aligned}\underline{\dot{x}} &= \underline{A}\underline{x} + \underline{B}\underline{u}\\ \underline{y} &= \underline{C}\underline{x}\end{aligned} \tag{8}$$

where newly defined state vector, expanded with components of the back electromotive force was introduced:

$$\underline{x} = col\left(i_{sa}, i_{s\beta}, e_{sa}, e_{s\beta}\right). \tag{9}$$

So the state and control matrices as well as the control vector are the following:

$$A = \begin{bmatrix} -\frac{R_s}{L_s} & 0 & -\frac{1}{L_s} & 0 \\ 0 & -\frac{R_s}{L_s} & 0 & -\frac{1}{L_s} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} \frac{1}{L_s} & 0 \\ 0 & \frac{1}{L_s} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{10}$$

$$\underline{u} = col\left(u_{sa}, u_{s\beta}, 0, 0\right) \tag{11}$$

Using the Luenberger theory of state observation [7], it is possible to design a reduced-order state observer [9]. The mathematical model of this observer for the back electromotive force components takes the following form:

$$\frac{d}{dt}\underline{\varepsilon_s} = g \cdot \left[\underline{\varepsilon_s} - \underline{u_s} + \left(g \cdot \underline{L_s} + \underline{R_s}\right) \cdot \underline{i_s}\right] \tag{12}$$

$$\underline{\hat{e}_s} = \underline{\varepsilon_s} + \left(g \cdot \underline{L_s}\right) \cdot \underline{i_s} \tag{13}$$

where $\underline{\varepsilon_S}$ – is an auxiliary variable, g – an eigenvalue of the reduced-order state observer chosen so, that the state matrix of the observer is of special form:

$$\underline{F} = diag\left(g, g\right), \tag{14}$$

with $$g \langle \lambda_i \langle 0 \tag{15}$$

where λ_i – are motor eigenvalues.

Based on the estimation of back electromotive force it is possible to reconstruct the sine and cosine functions of the rotor angular position as well as the motor speed:

$$\sin\hat{\Theta} = \frac{\hat{e}_{s\alpha}}{\left|\hat{e}_s\right|} \quad ; \quad \cos\hat{\Theta} = \frac{\hat{e}_{s\beta}}{\left|\hat{e}_s\right|} \tag{16}$$

$$\left|\hat{\omega}\right| = \frac{\left|\hat{e}_s\right|}{k_e} \tag{17}$$

It was shown in the paper [11], that the reduced-order state observer is practically robust to stator parameters changes (R_s, L_s). However the quality of the estimation depends on precise identification of k_e coefficient. Therefore in this paper the new observer was proposed, which reconstructs not only rotor speed but also the motor voltage constant k_e.

3.3.The extended full-order observer

After the transformation of the PMSM mathematical model (1)-(3) to the matrix state equation, where the unknown parameter k_e was added to the system state vector, an extended full-order state observer can be design [10]. The new extended state model of the motor is the following:

$$\begin{aligned}\underline{\dot{x}} &= \underline{A}_r \underline{x}_r + \underline{B}_r \underline{u}\\ \underline{y} &= \underline{C}_r \underline{x}_r\end{aligned} \tag{18}$$

where:

- extended state vector of the system:

$$\underline{x}_r = col\left(i_{sa}, i_{s\beta}, \omega, k_e\right) \tag{19}$$

- the extended state, control and output matrices, respectively:

$$A_r = \begin{bmatrix} -\frac{R_s}{L_s} & 0 & -\frac{\sin\Theta}{L_s}k_e & -\frac{\sin\Theta}{L_s}\omega \\ 0 & -\frac{R_s}{L_s} & -\frac{\cos\Theta}{L_s}k_e & -\frac{\cos\Theta}{L_s}\omega \\ \frac{\sin\Theta}{J}k_i & \frac{\cos\Theta}{J}k_i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{20}$$

$$B_r = \begin{bmatrix} \frac{1}{L_s} & 0 \\ 0 & \frac{1}{L_s} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{21}$$

$$C_r = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{22}$$

and the control vector is as (11).

Based on these equations the extended full-order state observer can be designed, according to [10]:

$$\begin{aligned}\dot{\hat{i}}_{s\alpha} &= -\frac{R_s}{L_s}\hat{i}_{s\alpha} - \frac{\sin\hat{\Theta}}{L_s}\hat{k}_e\hat{\omega} + \frac{1}{L_s}u_{s\alpha} + \\ &\quad + k_{11}\left(i_{s\alpha} - \hat{i}_{s\alpha}\right) + k_{12}\left(i_{s\beta} - \hat{i}_{s\beta}\right)\\ \dot{\hat{i}}_{s\beta} &= -\frac{R_s}{L_s}\hat{i}_{s\beta} - \frac{\cos\hat{\Theta}}{L_s}\hat{k}_e\hat{\omega} + \frac{1}{L_s}u_{s\beta} + \\ &\quad + k_{21}\left(i_{s\alpha} - \hat{i}_{s\alpha}\right) + k_{22}\left(i_{s\beta} - \hat{i}_{s\beta}\right)\\ \dot{\hat{\omega}} &= \frac{\sin\hat{\Theta}}{J}\hat{k}_e i_{s\alpha} + \frac{\cos\hat{\Theta}}{J}\hat{k}_e i_{s\beta} + T_e + \\ &\quad + k_{31}\left(i_{s\alpha} - \hat{i}_{s\alpha}\right) + k_{32}\left(i_{s\beta} - \hat{i}_{s\beta}\right)\\ \dot{\hat{k}}_e &= k_{41}\left(i_{s\alpha} - \hat{i}_{s\alpha}\right) + k_{42}\left(i_{s\beta} - \hat{i}_{s\beta}\right)\end{aligned} \tag{23}$$

where k_{ij} are elements of the gain matrix of $\underline{K}$, and all variables denoted by "$\wedge$" are estimated ones.

4.SIMULATION RESULTS

All described methods of speed and position estimation were tested in simulations. The influence of changing stator parameters R_s, L_s and motor constant k_e to the estimation quality was checked for all estimators working in the open-loop. During these tests, the actual rotor speed and position were "measured" in the motor and used not only for closed-loop control in the drive system structure but also for the comparison with estimated variables. Simulation tests were performed for the structure presented in Fig.2, in Matlab-Simulink.

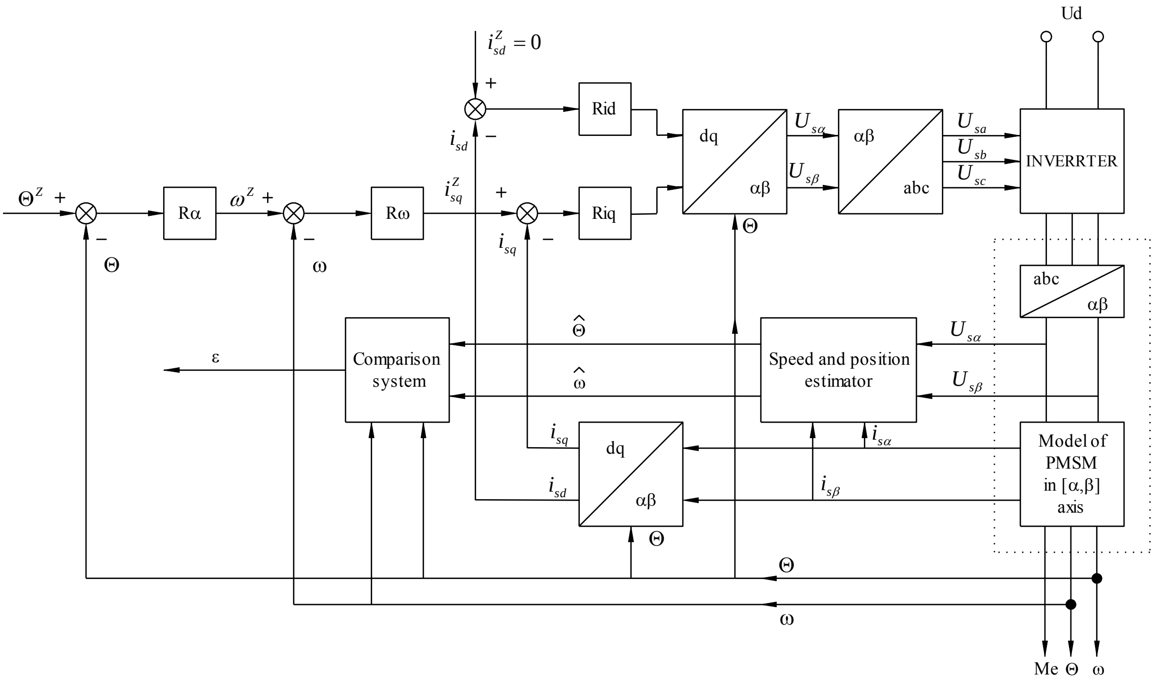

Fig.2 Schematic diagram of the vector-controlled PMSM drive system with speed and position estimators

The rotor speed reference was changed as shown in Fig.3, with the assumption that the motor was loaded by the torque $T_L=3T_{eN}$, the drive system inertia was equal to the motor inertia $J_o=J_N$ and maximal allowed stator current was limited to the four nominal motor current value $I_{mac}=4I_N$. Simulations were done for the drive system with PMSM motor LX440, with the following parameters:
I_N=14,8[A], n_N=4480[rev/min], T_{eN}=6,7[Nm], J_N=0,00039[kgm^2], k_e=62,5/1000[Vmin/rev], k_i=0,45[Nm/A], R_s=0,78[Ω], L_s=5,3[mH].

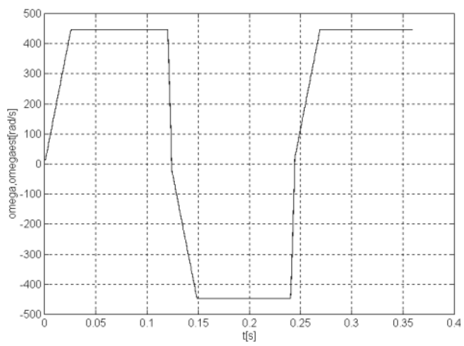

Fig.3 The rotor speed reference

All simulation tests were performed in per unit system. The estimation errors obtained from the speed and position estimators and observers were calculated in the following way:
for relative errors:

$$\begin{aligned}\Delta\omega &= \omega - \hat{\omega}\ [\text{rad/s}]\\ \Delta\Theta &= \Theta - \hat{\Theta}\ [\text{rad}]\end{aligned} \tag{24}$$

- for the absolute errors:

$$\begin{aligned}\delta\omega &= \frac{|\omega - \hat{\omega}|}{\omega_n}\cdot 100\,[\%]\\ \delta\Theta &= \frac{|\Theta - \hat{\Theta}|}{\Theta_{\max}}\cdot 100\,[\%]\end{aligned} \tag{25}$$

It was impossible to calculate the absolute errors in reference to the actual speed and position value, because of the assumed speed reference track, which has contained the zero-crossing values of speed reference.
In the following figures some examples of absolute speed estimation errors, calculated according to (25), were shown for all tested estimators, in the case of the same changes of stator resistance (+50%) and stator inductance (–50%). So, in Fig.4 the speed reconstruction errors for the method based directly on equations of the motor (Eq.(5)-(7)) are presented, in Fig.5 - for the method based on the reduced-state observer of Luenberger type (Eq.(12)-(17)) and respectively in Fig.6 - for the method based on the full-order extended state observer (Eq.(23)).
From Fig.4 concludes, that for the speed estimator based directly on motor equations, transient errors are very big, in the range of 200% for respective stator resistance changes and in the range of 20% for the inductance changes. Much better results were obtained for the method based on the reduced-order state observer. For this observer transient errors of speed estimation are 10% and 14% for the respective changes of motor resistance and inductance. The best results were obtained for the method based on the full-order extended state observer. In this case the speed estimation errors are not bigger than 3%.

a) R_s=150%R_{sN}, L_s=L_{sN} b) R_s=R_{sN}, L_s=50%L_{sN}

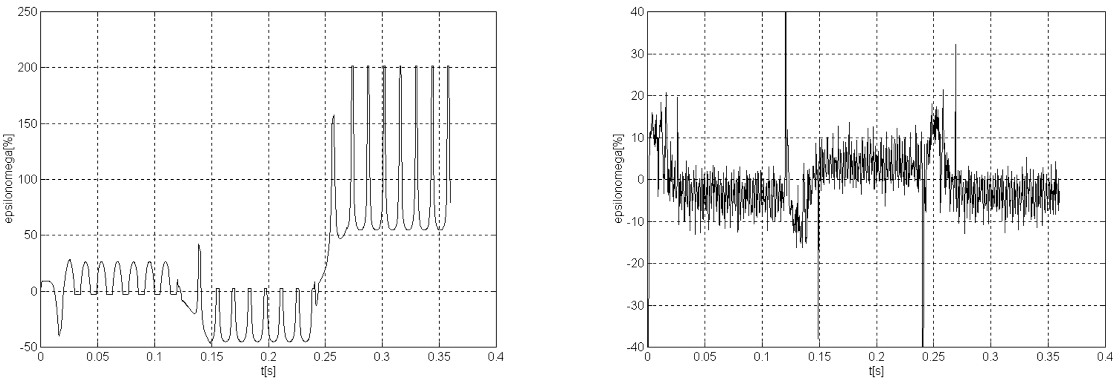

Fig.4 Absolute speed estimation errors for the estimator based directly on motor equations

a) R_s=150%R_{sN}, L_s=L_{sN} b) R_s=R_{sN}, L_s=50%L_{sN}

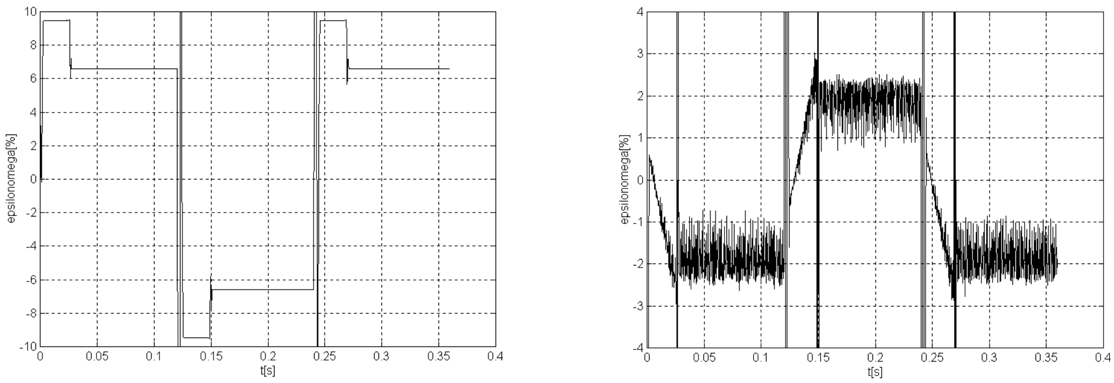

Fig.5 Absolute speed estimation errors for the estimator based on the reduced-order state observer

a) R_s=150%R_{sN}, L_s=L_{sN} b) R_s=R_{sN}, L_s=50%L_{sN}

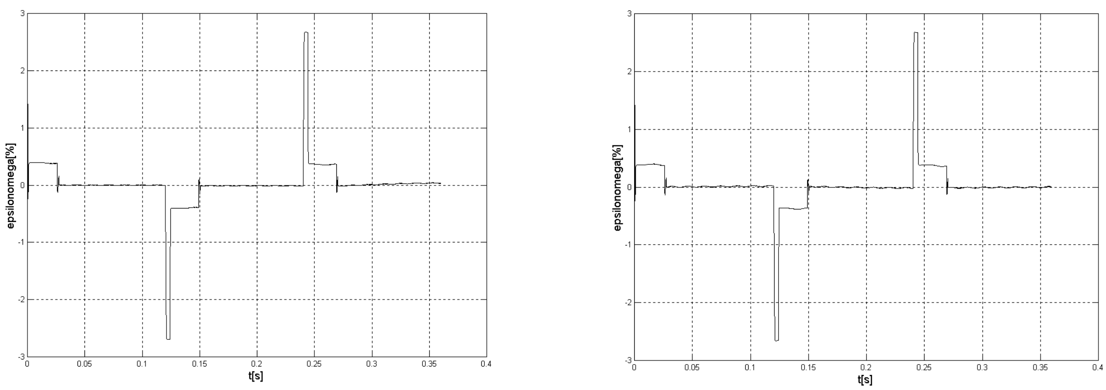

Fig.6 Absolute speed estimation errors for the estimator based on the full-order extended state observer

In the next figures the comparison of average absolute errors of speed estimation for all three estimation methods was shown: in Fig.7 – for stator resistance changes and in Fig.8 – for the stator inductance changes. In a similar way the comparison of average absolute estimation errors of the rotor angular position was presented. In all figures the following description was used:

S1 – method based directly on the motor equation,

S2 – method based on the reduced-order state observer,

S3 – method based on the full-order state observer.

From the comparison of average speed estimation errors for all three methods (Fig.7, 8) it is seen, that the biggest errors occur for the method based directly on the motor equations: these errors are in the range of 40% for ±50% stator resistance changes and in the range of 6% for ±50% stator inductance changes. For the estimation method based on the reduced-order state observer the average errors are respectively in the range of 5% and 3%. The best results were obtained for the full-order extended state observer: the average errors of speed estimation were smaller than 1% for both stator resistance and inductance change.

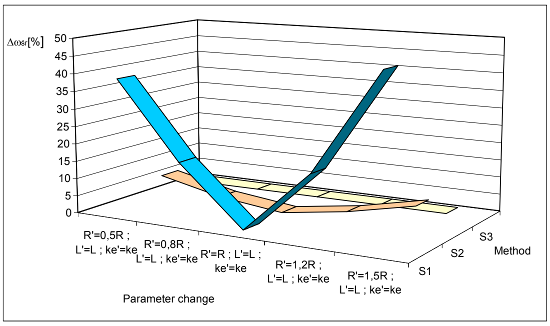

Fig.7 Average speed errors for stator resistance changes

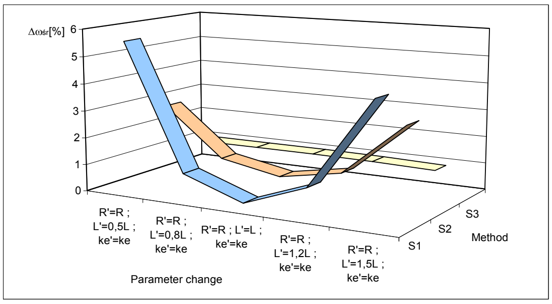

Fig.8 Average speed errors for stator inductance changes

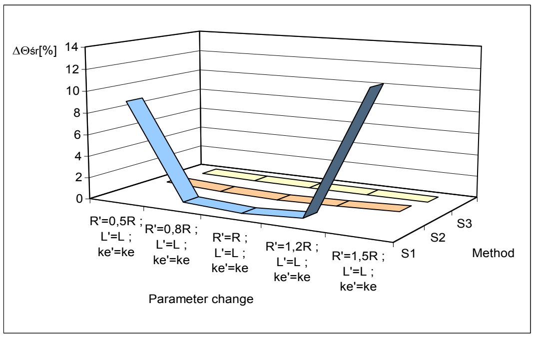

Fig.9 Average position errors for stator resistance changes

Respectively, the biggest average position errors occur for the estimator based directly on motor equations – 10% for stator resistance changes and about 1% for the stator inductance changes (as shown in Fig.9, 10). For the method based on the reduced-order state observer these errors are in the range of 1% for stator resistance changes and 0,5% for stator inductance changes. In this case the best results were obtained for the extended full-order state observer: estimation errors are smaller than 0,2%, both for resistance and inductance changes.

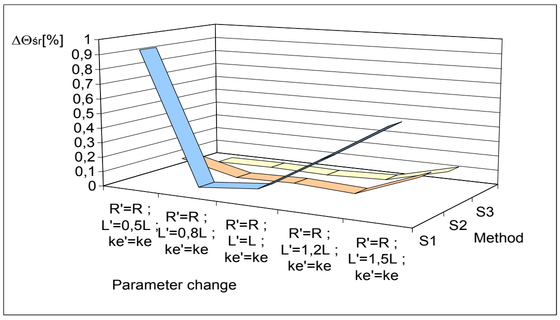

Fig.10 Average position errors for stator inductance changes

Additionally, for methods based on the reduced-order and the extended full-order state observers, the effect of change of motor voltage constant k_e for speed estimation errors was checked. Results were shown in Fig.11.

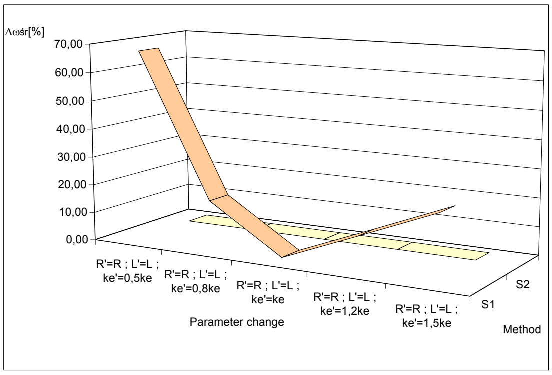

Fig.11 Average speed errors for motor constant k_e changes

It is seen from this figure that for the reduced-order state observer any change of k_e parameter cause the significant increase of speed estimation error. However, the extended full-order state observer is almost completely robust for change of motor voltage constant. For this method the average speed estimation error was not bigger than 1%, when for the same k_e change in the case of the reduced-order observer this error was in the range of 70%. So the additional estimation of actual value of voltage constant of the motor has reduced significantly the speed and position estimation errors.

5.SUMMARY

In the paper three estimation methods for PMSM speed and position reconstruction were described and evaluated based on the comparison of transient and average estimation errors calculated during the different operation modes of the vector-controlled drive system.

From performed simulation tests for PMSM speed and position estimation methods can be concluded, that the realisation of the PMSM drive system without mechanical sensors with good dynamic and static requirements is possible. The biggest sensitivity for change of motor parameters (L_s, R_s) was found for the method based directly on motor equation. Better results were obtained for method based on the reduced-order state observer and the best results were obtained for the method based on the extended full-order state observer.

Additionally, the extended full-order state observer is practically robust for changes of motor voltage constant k_e. Opposite, in the case of the reduced-order speed observer even little change of k_e constant causes a significant increase of the speed estimation error.

If a temperature correction of the stator resistance is introduced to the control system, the method of speed and position estimation based on the extended full-order observer proposed in this paper seems to be an interesting solution for the sensorless drive system with permanent magnet synchronous motor.

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BIOGRAPHIES

Teresa Orlowska-Kowalska, Ph.D., D.Sc., prof., has obtained her scientific degrees in 1976, 1990 respectively, at Electrical Engineering Faculty of Wroclaw University of Technology. Since 1993 she has the professor's position at this faculty and is head of Electrical Drives Control Chair in Institute of Electrical Machines and Drives. She is author and co-author of over 100 scientific papers. Her field of interest is the mathematical modelling and microprocessor control of electrical drives and power electronic systems, application of modern control methods to electrical drives, state estimation of induction motors with help of observers and neural networks. In the last years her main field of research is connected with neural networks and fuzzy-logic techniques applied to electrical drives control. She is the member of Electrical Engineering Committee of Polish Academy of Science, IEEE, IMACS, EPE, CIGRE and member of the international steering committees of few well-known European conferences.

Marek Lec, M.Sc., has finished his Master Thesis in 1997 and since this year he is Ph.D. student in the Institute of Electrical Machines, Drives and Measurements in Wroclaw University of Technology. His main field of interest is the application of PMSM in the electrical drives, power electronics and microprocessor techniques.