SOME INTERESTING APPLICATIONS OF THE KARNAUGH MAP

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SUMMARY
The article describes a construction of a Boolean (Zhegalkin) polynomial with respect to a given Boolean function, and vice versa. It also presents a simple way of writing symmetric Boolean functions that allow easy construction of minimal disjunctive normal forms.

Keywords: *Boolean function, symmetric Boolean function, Shannon expansion, Reed-Muller expansion, Boolean polynomial, Karnaugh map*

1. INTRODUCTION

Binary logical structural models are usually written as friendly normal disjunctive formulae of output Boolean functions. It often is to advantage to apply Boolean (Zhegalkin) polynomials, whose writing with respect to the given Boolean functions is not always simple.

Among Boolean functions a significant role is played (a significant position is held) by completely symmetric (further only:symetric) functions or their special cases – special threshold or majority functions, whose specific features, if they act as output functions, ensure simple modelling of combination circuits.

2. A NORMAL DISJUNCTIVE FORMULA AND BOOLEAN POLYNOMIALS

Be given a total Boolean function $f(x_1, x_2, \ldots, x_m)$, m of arguments

$$f:\{0,1\}^m \to \{0,1\}: \langle x_1, x_2, \ldots, x_m \rangle \mapsto y \tag{1}$$

It can be easily shown (through generalising its Shannon development) that each Boolean function $f(x_1, x_2, \ldots, x_m)$ is represented by a complete normal (canonical) disjunctive formula (***cndf***)

$$f(x_1, x_2, \ldots, x_m) = \bigvee_{\langle \sigma_1, \sigma_2, \ldots, \sigma_m \rangle} f(\sigma_1, \sigma_2, \ldots, \sigma_m)\, x_1^{\sigma_1} x_2^{\sigma_2} \ldots x_m^{\sigma_m} \tag{2}$$

where $\vee$ is a disjunction operator and $x^{\sigma} = x\,\sigma \vee \overline{x}\,\overline{\sigma}$, resp.

$$x^{\sigma} = \begin{cases} \overline{x} & \text{for } \sigma = 0 \\ x & \text{for } \sigma = 1 \end{cases} \tag{3}$$

The given Boolean function can, however, be expressed [1,2] by a complete normal formula according to modulo 2

$$f(x_1, x_2, \ldots, x_m) = \bigoplus_{\langle \sigma_1, \sigma_2, \ldots, \sigma_m \rangle} f(\sigma_1, \sigma_2, \ldots, \sigma_m)\, x_1^{\sigma_1} x_2^{\sigma_2} \ldots x_m^{\sigma_m} \tag{4}$$

where $\oplus$ is a operator of modulo 2 sum, because the conjunction of arbitrary, mutually different minterms $x_1^{\sigma_1} x_2^{\sigma_2} \ldots x_m^{\sigma_m}$ is always equal to 0. It can be also expressed by a Boolean (Zhegalkin) polynomial [2,3]

$$f(x_1, x_2, \ldots, x_m) = \bigoplus_{\langle \sigma_m, \sigma_{m-1}, \ldots, \sigma_1 \rangle} r(\sigma_m, \sigma_{m-1}, \ldots, \sigma_1)\, x_1^{\sigma_1} x_2^{\sigma_2} \ldots x_m^{\rho_m} \tag{5}$$

where $x^{\sigma} = x \vee \overline{\sigma}$, resp.

$$x^{\sigma} = \begin{cases} 1 & \text{for } \sigma = 0 \\ x & \text{for } \sigma = 1 \end{cases} \tag{6}$$

Let us now get concerned with writing a Boolean polynomial to a given Boolean function, or with transforming **c*ndf*** to the Zhegalkin polynomial, and *vice versa* . Note that if $M = [m_{ij}]$ and $\mathcal{M} = [m_{jk}]$ are Boolean matrices having the respective dimensions $m \times n$ a $n \times o$, by their Cartesian product $M \times \mathcal{M} = [c_{ik}]$ a matrix of $m \times o$ is meant such that $c_{ik} = \bigoplus_j m_{ij}\, m_{jk}$.

On the one hand let us consider first the Shannon development (***cndf)*** of the function $f(x)$, i.e., $f(x) = f(0)\overline{x} \vee f(1)x$; since $\overline{x} = 1 \oplus x$, the so called Reed and Muller expansion of the function $f(x)$, i.e.,

$$f(x) = f(0)(x \oplus 1) \oplus f(1)x = f(0) \oplus (f(0) \oplus f(1)) \tag{7}$$

and on the other hand the Zhegalkin polynomial of the function $f(x)$, i.e.,

$$f(x) = r(0) \oplus r(1)x\,. \tag{8}$$

Hence in a matrix form it is [3] :

$$\begin{bmatrix} r(0) \\ r(1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} f(0) \\ f(1) \end{bmatrix} \tag{9}$$

Through analogy we obtain for $f(x_1, x_2)$ on the one hand:

$$f(x_1, x_2) = f(0,0) \oplus (f(0,0) \oplus f(1,0))\, x_1 \oplus$$
$$\oplus (f(0,0) \oplus f(0,1)) x_2 \oplus$$
$$\oplus (f(0,0) \oplus f(0,1) \oplus f(1,0) \oplus f(1,1))\, x_1 x_2 \quad (10)$$

on the other hand :

$$f(x_1, x_2) = r(0) \oplus r(1) x_1 \oplus r(2) x_2 \oplus r(3) x_1 x_2 \ .$$

In another form, [3] :

$$\begin{bmatrix} r(0) \\ r(1) \\ r(2) \\ r(3) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} f(0,0) \\ f(1,0) \\ f(0,1) \\ f(1,1) \end{bmatrix} = \begin{bmatrix} T_1 & \Theta \\ T_1 & T_1 \end{bmatrix} \times \begin{bmatrix} f(0,0) \\ f(1,0) \\ f(0,1) \\ f(1,1) \end{bmatrix} \quad (11)$$

where Θ and T_1 are the respective submatrices. In generel, [2,3], for $n \geq 1$

$$T_n = \begin{bmatrix} T_{n-1} & \Theta \\ T_{n-1} & T_{n-1} \end{bmatrix} \quad (12)$$

holds, when $T_0 = [1]$. On the contrary, for $f(x_1, x_2)$ $= = r(0) \oplus r(1) x_1 \oplus r(2) x_2 \oplus r(3) x_1 x_2$ we obtain $f(0,0) = r(0)$, $f(0,1) = r(0) \oplus r(2)$, $f(1,0) = r(0) \oplus r(1)$ and $f(1,1) = r(0) \oplus r(1) \oplus r(2) \oplus r(3)$, i.e.

$$\begin{bmatrix} f(0,0) \\ f(1,0) \\ f(0,1) \\ f(1,1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} r(0) \\ r(1) \\ r(2) \\ r(3) \end{bmatrix} \quad (13)$$

In other word, for on inverse T_n^{-1} transformation matrix the relationship $T_n^{-1} = T_n$ [2,3] is generally valid. The order of the matrix T_n appears to be 2^m.

The reader may already see that the transformation of ***cndf*** to the Zhegalkin polynomial and *vice versa* is no easy procedure for $n \geq 3$.

3. BOOLEAN POLYNOMIAL

For each number $\rho = 0,1,...,2^m - 1$, where ρ is a decadic equivalent of a binary number $\rho_m \rho_{m-1} \dots \rho_1 \left(\sum_{k=1}^{m} 2^{k-1} \rho_k \right)$, write $\mathcal{M}(\rho)$ for the set of all numbers π ($\pi \leq \rho$) in whose binary notation $\pi_m \pi_{m-1} \dots \pi_1 \left(= \sum_{k=1}^{m} 2^{k-1} \pi_k \right)$ 1s occur only in places in which they occur in the number ρ, i.e.,

$$\mathcal{M}(\rho) = \{ \pi_m \pi_{m-1} \dots \pi_1 \mid \pi_k = 1 \Rightarrow \rho_k = 1 \} \quad (14)$$

where $\Rightarrow$ is an implication operator; for instance. $\mathcal{M}(0) = \{0\}$, $\mathcal{M}(1) = \{0,1\}$, $\mathcal{M}(3) = \{0,1,2,3\}$. Now it is clear that the set $\mathcal{M}(\rho) = \mathcal{M}(\rho_m \rho_{m-1} \dots \rho_1)$ consists of all 0–cubes covered by a cube of the **least** dimension necessarily containing 0–cubes with coordinates 0 on the one hand and ρ on the other hand pertaining to the *m*–dimensional cube , which is the carrier of carries cube complex of the given function $f(x_1, x_2, ..., x_m)$. For instance the sets $\mathcal{M}(3)$, $\mathcal{M}(4)$, $\mathcal{M}(5)$, $\mathcal{M}(6)$ i $\mathcal{M}(7)$ can be easily represented by cubes on a 3-dimensional - map, such as the Karnaugh map (Fig. 1) so that $\mathcal{M}(3) = \{0,1,2,3,\}$, $\mathcal{M}(4) = \{0,4\}$, $\mathcal{M}(5) = \{0,1,4,5\}$, $\mathcal{M}(6) = \{0,2,4,6\}$ and $\mathcal{M}(7) = \{0,1, ..., 7\}$. Geometrical representation of Boolean functions is discussed, e.g., in [4]. The dimension of the *m*-dimensional Karnaugh map equals to $2^k \times 2^l$ ($k+l = m$), whereas the dimension of the transformation matrix T_n is 2^{2m}.

Now let us present that a Boolean polynomial representing a given total Boolean function can be expressed by the sets $\mathcal{M}(\rho)$.

Each Boolean total function $f(x_1, x_2, ..., x_m)$ can be expressed [5] by the Zegalkin polynomial

$$f(x_1, x_2, \dots, x_m) =$$
$$= \bigoplus_{\langle \rho_m, \rho_{m-1}, \dots, \rho_1 \rangle} r(\rho_m, \rho_{m-1}, \dots, \rho_1)\, x_1^{\rho_1} x_2^{\rho_2} \dots x_m^{\rho_m} \quad (15)$$

where

$$r(\rho) = \bigoplus_{\rho_m \rho_{m-1} \dots \rho_1 \in \mathcal{M}(\rho)} f(\rho_1, \rho_2, \dots, \rho_m) \quad (16)$$

Indeed, through mathematical induction for $m = 1$ we obtain

$$f(x) = r(0) \oplus r(1) x = f(0) \oplus (f(0) \oplus f(1)) x \quad (17)$$

and for $f(x_1, x_2, ..., x_m, x_{m+1})$ we obtain

$$f(x_1, x_2, ..., x_m, x_{m+1}) = f(x_1, x_2, ..., x_m, 0) \oplus$$
$$\oplus (f(x_1, x_2, ..., x_m, 0) \oplus f(x_1, x_2, ..., x_m, 1)\, x_{m+1} \quad (18)$$

Since both equations are valid:

$$f(x_1, x_2, ..., x_m, 0) =$$
$$= \bigoplus_{\langle 0, \rho_m, \rho_{m-1}, \dots, \rho_1 \rangle} r(0, \rho_m, \rho_{m-1}, \dots, \rho_1) x_1^{\rho_1} x_2^{\rho_2} \dots x_m^{\rho_m} 0^0$$
$$= \bigoplus_{\langle \rho_m, \rho_{m-1}, \dots, \rho_1 \rangle} r(\rho_m, \rho_{m-1}, \dots, \rho_1)\, x_1^{\rho_1} x_2^{\rho_2} \dots x_m^{\rho_m} \quad (19)$$

where

$$r(\rho) = \bigoplus_{0\, \rho_m \rho_{m-1} \dots \rho_1 \in \mathcal{M}(\rho)} f(\rho_1, \rho_2, \dots, \rho_m, 0) =$$
$$= \bigoplus_{\rho_m \rho_{m-1} \dots \rho_1 \in \mathcal{M}(\rho)} f(\rho_1, \rho_2, \dots, \rho_m) \quad (20)$$

and

$$f(x_1, x_2, ..., x_m, 1) =$$
$$= \bigoplus_{\langle 1, \rho_m, \rho_{m-1}, \dots, \rho_1 \rangle} \mathrm{r}(1, \rho_m, \rho_{m-1}, \dots, \rho_1) .$$
$$. x_1^{\rho_1} x_2^{\rho_2} \dots x_m^{\rho_m} 1^1 =$$
$$= \bigoplus_{\langle 1, \rho_m, \rho_{m-1}, \dots, \rho_1 \rangle} r(1, \rho_m, \rho_{m-1}, \dots, \rho_1) .$$
$$. x_1^{\rho_1} x_2^{\rho_2} \dots x_m^{\rho_m} \quad (21)$$

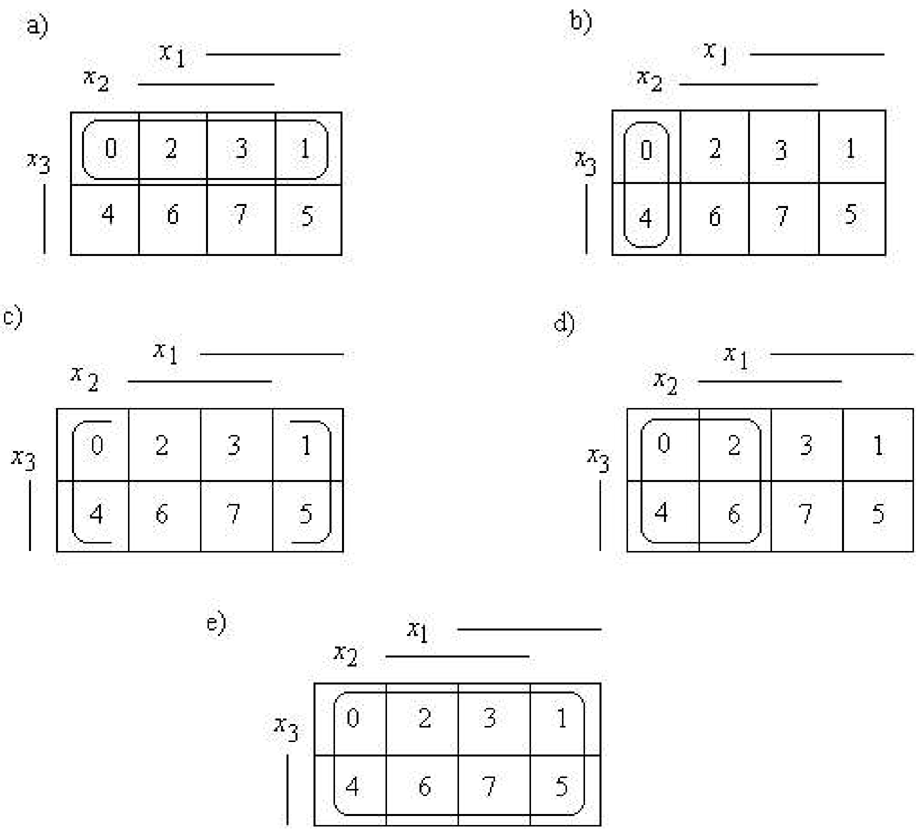

Fig. 1 Karnaugh maps of the sets: a) $\mathcal{M}(3)$, b) $\mathcal{M}(4)$, c) $\mathcal{M}(5)$, d) $\mathcal{M}(6)$, e) $\mathcal{M}(7)$

where

$$r\left(\rho+2^m\right) = \bigoplus_{1\,\rho_m\rho_{m-1}\ldots\rho_1\in \mathcal{M}\left(\rho+2^m\right)} f\left(\rho_1,\rho_2,\ldots,\rho_m,1\right) \tag{22}$$

it can be also said that

$$f(x_1, x_2, \ldots, x_m, x_{m+1}) = \bigoplus_{\rho} r(\rho)\, x_1^{\rho_1} x_2^{\rho_2} \ldots x_m^{\rho_m} \oplus$$
$$\oplus \left(\bigoplus_{\rho} r(\rho) x_1^{\rho_1} x_2^{\rho_2} \ldots x_m^{\rho_m} \oplus \bigoplus_{\rho+2^m} r\left(\rho+2^m\right) x_1^{\rho_1} x_2^{\rho_2} \ldots x_m^{\rho_m} \right) x_{m+1} \tag{23}$$

which is the Reed-Muller exppansion of the function $f(x_1, x_2, \ldots, x_m, x_{m+1})$.

<u>Example 1.:</u> Write the Boolean function y = 11000111 by applying the Zegalkin polynomial. Therefore we will write the given function into the Karnaugh map:

	$\bar{x}_1\bar{x}_2$	$\bar{x}_1 x_2$	$x_1 x_2$	$x_1\bar{x}_2$
$\bar{x}_3$	0: 1	2: 0	3: 0	1: 1
x_3	4: 0	6: 1	7: 1	5: 1

and hence:

$y = r(0) \oplus r(1)\, x_1 \oplus r(2)\, x_2 \oplus r(3)\, x_1x_2 \oplus r(4)\, x_3 \oplus$
$\oplus\, r(5)\, x_1x_3 \oplus r(6)\, x_2x_3 \oplus r(7)\, x_1x_2x_3 =$
$= 1 \oplus (1 \oplus 1)x_1 \oplus (1 \oplus 0)x_2 \oplus (1\oplus 0 \oplus 1 \oplus 0)\, x_1x_2 \oplus$
$\oplus\, (1 \oplus 0)\, x_3 \oplus (1 \oplus 1 \oplus 0 \oplus 1)\, x_1x_3 \oplus$
$\oplus\, (1 \oplus 0 \oplus 0 \oplus 1)\, x_2x_3 \oplus (1 \oplus 1 \oplus 0 \oplus 0 \oplus$
$\oplus\, 0 \oplus 1 \oplus 1 \oplus 1)\, x_1x_2x_3 = 1 \oplus x_2 \oplus x_3 \oplus x_1x_3 \oplus x_1x_2x_3.$

which could also be written directly. ■

<u>Example 2.:</u> A function is to be found to the Zegalkin polynomial $y = 1 \oplus x_1 \oplus x_1x_2 \oplus x_1x_2x_3$ expressed by the given Zhegalkin polynomial itself. Since $r(0) = r(1) = r(3) = r(7) = 1$ as well as $r(2) = r(4) = r(5) = r(6) = 0$, it can be written

$r(0) = 1 \Rightarrow f(0,0,0) = 1,$
$r(1) = 1 \Rightarrow f(0,0,0) \oplus f(1,0,0) = 1 \Rightarrow$
$\Rightarrow 1 \oplus f(1,0,0,) = 1 \Rightarrow f(1,0,0,) = 0$
$r(2) = 0 \Rightarrow f(0,0,0) \oplus f(0,1,0) = 0 \Rightarrow$
$\Rightarrow 1 \oplus f(0,1,0) = 0 \Rightarrow f(0,1,0) = 1$
$r(3) = 1 \Rightarrow f(0,0,0) \oplus f(1,0,0) \oplus f(0,1,0) \oplus$
$\oplus f(1,1,0) = 1 \Rightarrow 0 \oplus 0 \oplus 1 \oplus f(1,1,0) =$
$= 1 \Rightarrow f(1,1,0,) = 1,$

and so on … This can be easier and more conveniently seen from the Karnaugh map, into which we successively write 0 or 1, attempting to satisfy the required values $r(\rho_3, \rho_2, \rho_1)$:

0: 1	2: 1	3: 1	1: 0
4: 1	6: 1	7: 0	5: 0

Hence y = 10111010 ■

4. SYMETRIC FUNCTIONS

A Boolean function $y = f(x_1, x_2, ..., x_m)$ is called (totally) **symmetric** if it does not change its value y in an arbitrary permutation of its arguments $x_1, x_2, ..., x_m$ [2,6,7]. A Boolean function can also be called partly symmetric if there exists at least one subset containing at least two arguments. For this subset the function is completely symmetric. In other words, a function f is symmetric - $f(x_1, x_2, ..., x_m) = S_P^m\ (x_1, x_2, ..., x_m) = S_P^m$ - if there exists a set P ($P = \{p_i\}_{i=1}^{k}$) of operating, (non-negative integer) numbers p_i ($p_i \in \{j\}_{j=0}^{m}$) pro $k = 0,1,...,m$ such that the function f assumes unary value on all ordered Boolean m-tuple values of its arguments containing p_i of 1s; evidently $S_\varnothing^m = 0$ and $S_{\{0,1,...,m\}}^m = 1$.

$S_{\{0,1,...,m\}}^m$ can be expressed by using the disjunction of elementary symmetric functions $S_{\{p_i\}}^m$, i.e.

$$S_{\{p_1,p_2,...,p_k\}}^m = \bigvee_{i=1}^{k} S_{\{p_i\}}^m \tag{24}$$

The symetric function having at its disposal a single operating number is an *elementary* function. Since $S_P^m \vee S_Q^m = S_{P\cup Q}^m$, any symmetric function

5. ENTRY OF SYMETRIC FUNCTIONS

By Hamming's distance $d_H(a_1 a_2 ... a_m, b_1 b_2 ... b_m)$ of ordered Boolean m-tuples $\left(a_1 a_2 .. a_m, b_1 b_2 .. b_m \in \{0,1\}^m\right)$ we mean the distance $\sum_{j=1}^{m}(a_j \oplus b_j)$, and by Hamming's weight w_H ($a_1\ a_2\ ...a_m$) we mean the distance d_H ($a_1\ a_2\ ...a_m$, ,00...0) $= \sum_{j=1}^{m}(a_j \oplus 0) = \sum_{j=1}^{m} a_j$.

Writing each elementary symmetric function $S_{\{p_i\}}^m$ into the m-dimensional Karnaugh map without using a complete normal disjunctive formula which represents the particular function is relatively easy because there is a correspondence between all unary values of the function $S_{\{p_i\}}^m$ and the vertices (0-cubes) of the particular cube complex containing $\binom{m}{p_i}$ 0-cubes of Hamming's weight p_i.

<u>**Example 3.:**</u> Write into a Karnaugh map the function $S_{\{3\}}^5$ - Fig. 2, or $S_{\{3\}}^5$ assumes a unary value in $\binom{5}{2} = 10$ vertices of the particular cube complex whose distance is 3, or 5 – 3 = 2, from the vertex 000 00, or 111 11, respectively. ■

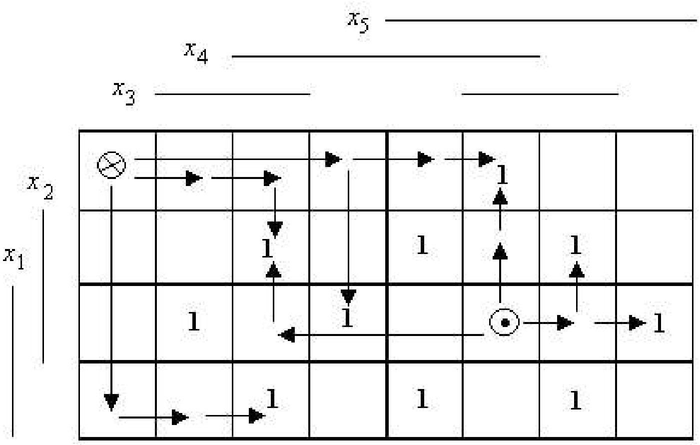

Fig. 2 Karnaugh map of the function $S_{\{3\}}^5$ from Example 3

Note, that a complete normal disjunctive formula (***cndf***) is the same as a minimal normal disjunctive formula (***mndf***) of the given elementary symetric function. Since each symmetric function can be expressed by the disjunction of the corresponding elementary symmetric functions, it is advantageous to use a Karnaugh map for writing the given symmetric function. In this way, we can easily arrive at the ***mndf*** of the given function, which is a markedly simple procedure than that from [1].

Example 4.: Construct ***mndf*** symmetric functions $S^4_{\{0,1,3,4\}}$. Because $S^4_{\{0,1,3,4\}} = S^4_{\{0\}} \bigvee S^4_{\{1\}} \bigvee S^4_{\{3\}} \bigvee S^4_{\{4\}}$, it may be easy the function $S^4_{\{0,1,3,4\}}$ written in Karnaugh map (Fig. 3) and written the minimal function ***mndf***:

$$S^4_{\{0,1,3,4\}} = \bar{x}_1\bar{x}_2\bar{x}_3 \vee \bar{x}_1\bar{x}_2\bar{x}_4 \vee \bar{x}_1\bar{x}_3\bar{x}_4 \vee \bar{x}_2\bar{x}_3\bar{x}_4 \vee$$
$$\vee x_1x_2x_3 \vee x_1x_2x_4 \vee x_2x_3x_4 \vee x_1x_3x_4$$

■

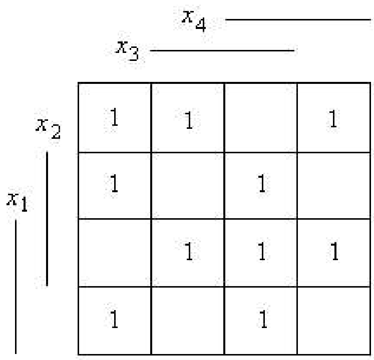

Fig. 3 Karnaugh map of function $S^4_{\{0,1,3,4\}}$ from Example 4

The Boolean function

$$f\left(x_1, x_2, ..., x_m\right) = sign\left(\sum_{j=1}^{m} w_j\, x_j - T\right), \qquad (24)$$

in which the weights w_j and the threshold T are real numbers is called a **threshold function.** If in the threshold function $w_i = w_j = w \ (i \neq j)$, we obtain the function

$$f\left(x_1, x_2, ..., x_m\right) = sign\left(\sum_{j=1}^{m} x_j - T/w\right). \qquad (25)$$

For some operating numbers p the following relationship holds $p - 1 < T/w \leq p$; then is $f(x_1, x_2, \ldots \ldots, x_m) = S^m_{\{p,p+1,\ldots,m\}}$, which is an elementary **voting** function [7]. Note [1] that only one (a single) ***mndf*** function $S^m_{\{p,p+1,\ldots,m\}}$ has the form of $\bigvee_{j=1}^{\binom{m}{p}} K_j$, where K_j is an elementary conjunct of length p with asserted variables. If, in addition, m is an odd number and $m \geq 3$ and $p = (m+1)/2$, then $f(x_1, x_2, ..., x_m)$ is a **majority** function.

Example 5.: Construct ***mndf*** of the function $sign\left(\sum_{j=1}^{4} x_j - 2\right) = S^4_{\{2,3,4\}}$. According to Fig. 4 we obtain

$$S^4_{\{2,3,4,\}} = x_1x_2 \vee x_1x_3 \vee x_1x_4 \vee x_2x_3 \vee x_2x_4 \vee x_3x_4$$

since $\binom{4}{2} = 6$. ■

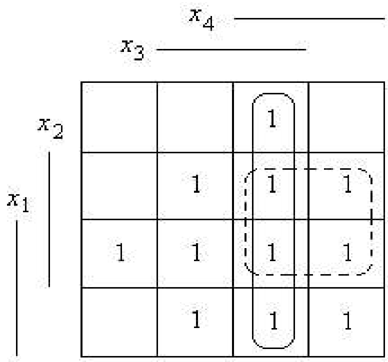

Fig. 4 Karnaugh map of the function $S^4_{\{2,3,4\}}$ from Example5

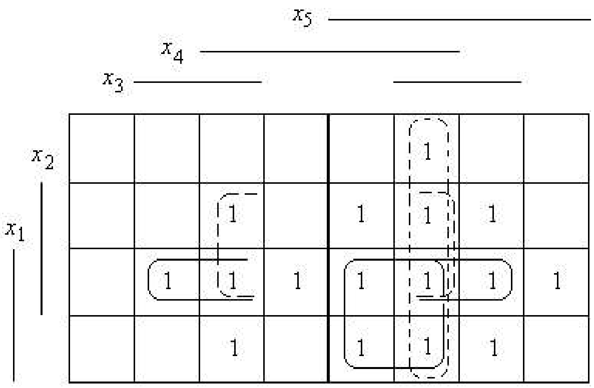

Fig. 5 Karnaugh map of the majority function from Example 6

Example 6.: Construct ***mndf*** of the majority function $Maj\left(x_1, x_2, x_3, x_4, x_5\right)$. Enter, therefore, the given function onto a Karnaugh map (Fig. 5), and because $Maj\left(x_1, x_2, x_3, x_4, x_5\right) = S^5_{\{3,4,5\}}$, we obtain

$$Maj\ (x_1, x_2, x_3, x_4, x_5) =$$
$$x_1x_2x_3 \vee x_1x_2x_4 \vee x_1x_2x_5 \vee x_1x_3x_4 \vee x_1x_3x_5 \vee$$
$$\vee\ x_2x_3x_4 \vee x_2x_3x_5 \vee x_2x_4x_5 \vee x_3x_4x_5\ ,$$

since $\binom{5}{3} = 10$ ■

For easier visualisation not all cubes of the cube complexes from Examples 5 and 6 are entered onto the maps in Figs 4 and 5.

6. CONCLUSIONS

The procedures of writing a given Boolean function by using the Zeghalkin polynomial and constructing the function represented by the polynomial can be regarded as sufficiently simple and convenient. This also applies to the design of ***mndf*** symmetrical Boolean functions, including the special cases of threshold and majority functions.

For finding Zeghalkin polynomials to the given Boolean functions and *vice versa*, as well as for finding the ***mndf*** symmetrical Boolean functions of m arguments $m \geq 7$, the Karnaugh map is not convenient, but Quine and Mc Cluskey method can be recommended.

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BIOGRAPHY

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