# USING VOLTAGE-DIP MATRICES FOR COUNTING OF VOLTAGE DIPS IN POWER SYSTEMS

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**SUMMARY**

*Voltage dips (sags), mainly caused by remote short circuits, are generally considered as the serious power quality problem. The most used calculation method for the prediction of the number of voltage dips in the power system is called the method of fault positions. The method is based on systematic calculation of short-circuit faults spread through the power system. By using short-circuit analysis algorithm the voltage dip matrices should be formed.*

*The dip-matrix contains the during-fault voltages (dip magnitudes) at each bus due to faults at each one of the buses and fault positions. The during-fault voltage at a general bus k when a fault occurs at that bus is contained in the diagonal of matrix. Off-diagonal matrix elements are the dips at a bus m due to a fault at a bus k. The column k contains the retained bus voltage for a fault at bus k, graphically presented by the affected area. The row m contains the retained voltages at that bus when faults occur at other buses, graphically presented by the exposed area or area of vulnerability. For more precise description of the exposed areas is needed to take into account the faults occurring on lines, so that number of columns of dip matrix increases with the number of possible fault positions. The expected number of dips and their characteristics can be determined by combining the dip matrix and the fault rates corresponding to the buses and the fault positions. For unsymmetrical faults the sequence dip matrices and consequently the dip matrices for each phase voltage should be formed. If we take the lowest phase voltage to characterize the dip magnitude, then each element of the final dip matrix is the lowest value of phase voltages, i.e. elements at corresponding row and column in the matrices for each phase voltage. Finally, four dip matrices characterize the bus voltages during three-phase, single-phase-to-ground, two-phase and two-phase-to-ground faults in the system respectively.*

*Matrix-based approach described in the paper is useful for computational implementation. It should be used not only for calculation of the expected number of voltage dip at chosen buses, but also for finding the optimal number of monitors and their emplacement in the power system.*

***Keywords:*** *power system, power quality, voltage dip, voltage sag, short-circuit fault, symmetrical components, impedance matrix, fault position*

## 1. INDRODUCTION

Voltage dips (sags) are short duration reductions in the rms voltage. Typically the dips are associated with the occurrence of short-circuit faults or other extreme increase in current like motor starting, transformer energizing, etc. Faults cause dips that can be observed far away from the fault location.

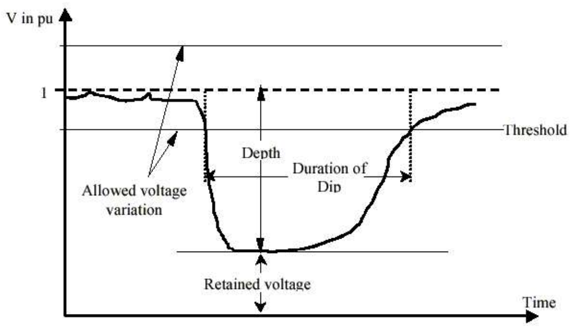


**Fig. 1** Characteristics of voltage dip

Voltage dips are characterized by their magnitude (the lowest rms voltage during the event) and duration (Fig. 1). In case of short-circuit faults the magnitude is determined by the electrical distance to the fault, the duration by the fault-clearing time. Dip magnitude ranges from 1% to 90% of nominal voltage. Most of dips usually last less than 1 s.

The effect of voltage dip on the operation of sensitive equipment can be the same like the effect of short supply interruption [1]. Statistical dip characterization of individual sites of a power network is essential to decide about mitigation methods. Statistics on voltage dip events (knowledge about occurrence and severity of voltage dips) may be obtained by using power quality monitoring or by means of prediction methods.

## 2. PREDICTION METHODS

The prediction methods are based on stochastic assessment. Stochastic assessment of voltage dips combines the stochastic data about the fault likelihood with the deterministic data regarding the remaining voltages during the fault.

The expected number of voltage dips at given site is found by using a model of the system and the statistics of short-circuit fault occurrence (fault rate of lines). The methods are as accurate as the model and the data used.

The most used method that can be used for the prediction of the number of voltage dips in the power system is method of fault positions [1]. The method is based on systematic calculation of short-circuit faults spread through the system. The position of a short-circuit fault is called the fault position (Fig. 2).

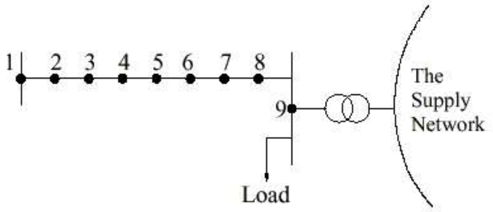


**Fig. 2** Fault positions

The remaining voltage (dip magnitude) is calculated for faults in each fault position (e.g. in a substation, along a line). Knowing the network parameters and the fault statistics of the network, the number of dips and distribution of remaining voltages can be estimated. An expected number of fault occurrences per year (fault rate) are allocated to each fault position. The voltage dip magnitude and duration at a given node are calculated for each fault position. After taking the failure rate for each fault into account, the expected number of dips as a function of magnitude and duration is calculated. This method is computationally simple and suitable for the computer implementation. The algorithm of the method is described in following sections in detail.

## 3. ANALYSIS OF FAULTED POWER SYSTEM

For obtaining statistics on magnitude of voltage dips in the buses, a modelling based on phasors is considered to be suitable technique. The use of phasors restricts the model to the context of steady state alternating linear systems, the modelling gives the during-fault voltage or the retained voltage during the fault and not the evolution as function of time.

For calculation of voltage dips and line-to-neutral voltages at all buses during the short-circuit fault, the bus impedance matrix $\mathbf{Z_{bus}}$ and the principle of superposition are used. The system is modelled by two circuits with opposite voltage sources $U_{0k}$ at the node $k$ with a fault [2]. The first one is circuit before occurrence of the fault, $U_{0k}$ is pre-fault voltage in node $k$. The second one is Thevenin equivalent circuit with voltage $-U_{0k}$ in node $k$, that draws the fault current $I_k$.

The voltages during the fault at node $k$ are represented by the vector $\mathbf{U_f}$ and expressed by matrix relation:

$$\mathbf{U_f} = \mathbf{U_0} - \mathbf{Z_{bus}} \cdot \mathbf{I_f} = \mathbf{U_0} + \mathbf{\Delta U_f} \tag{1}$$

where $\mathbf{Uo}$ is the vector of the pre-fault voltages, which are known for instance from a load-flow calculation, $\mathbf{\Delta U}$ is the vector of the changes in bus voltages due to the fault and the vector of the injected currents is $\mathbf{I_f} = [0 \ \cdots \ -I_k \ \cdots \ 0]^T$.

The use of the impedance matrix $\mathbf{Z_{bus}}$ provides a convenient means for calculating fault currents and voltages. The main advantage of this method is that once the bus impedance matrix is formed (by direct formation or by inverting the admittance matrix $\mathbf{Y_{bus}}$) the elements of this matrix can be used directly to calculate the currents and voltages associated with various types of faults.

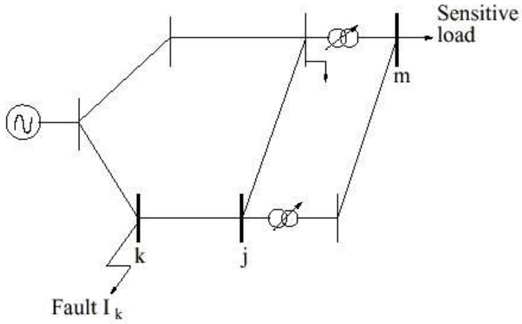


**Fig. 3** The system with a fault at node *k*

Equation (1) expressed by vectors shows, that only column $k$ of the impedance matrix $\mathbf{Z_{bus}}$ is needed to determine bus voltages during the fault at node $k$:

$$\begin{bmatrix} U_{f1} \\ \vdots \\ U_{fk} \\ \vdots \\ U_{fN} \end{bmatrix} = \begin{bmatrix} U_{01} \\ \vdots \\ U_{0k} \\ \vdots \\ U_{0N} \end{bmatrix} - \begin{bmatrix} Z_{1k} \cdot I_k \\ \vdots \\ Z_{kk} \cdot I_k \\ \vdots \\ Z_{Nk} \cdot I_k \end{bmatrix} = \begin{bmatrix} U_{01} \\ \vdots \\ U_{0k} \\ \vdots \\ U_{0N} \end{bmatrix} - \begin{bmatrix} Z_{1k} \cdot \frac{U_{0k}}{Z_{kk}} \\ \vdots \\ U_{0k} \\ \vdots \\ Z_{Nk} \cdot \frac{U_{0k}}{Z_{kk}} \end{bmatrix} \tag{2}$$

where the fault current for three-phase fault at node $k$ is given (fault impedance often is neglected)

$$I_k = \frac{U_{0k}}{Z_{kk}} \tag{3}$$

## 4. DIP MATRIX AND ITS INTERPRETATION

The equation (1) corresponds to the fault at bus $k$. The vector $\mathbf{U_f} = [U_{f1} \ \cdots \ U_{fk} \ \cdots \ U_{fN}]^T$, the vector of bus voltages, is $k^{th}$ column of the square matrix, called dip matrix. This matrix provides during-fault voltages at each bus and for the faults at each bus:

$$\begin{aligned} \mathbf{U_{dfv}} &= \mathbf{U_{pref}} - \mathbf{Z_{bus}} \cdot \mathbf{I} = \mathbf{U_{pref}} - \mathbf{\Delta U} \\ \mathbf{U_{dfv}} &= \mathbf{U_{pref}} - \mathbf{Z_{bus}} \cdot \mathbf{inv(diagZ_{bus})} \cdot \mathbf{U_{pref}^T} \end{aligned} \tag{4}$$

$\mathbf{U_{pref}}$ is the pre-fault voltage matrix and because the pre-fault voltage at node $k$ is the same for a fault at any node, the pre-fault voltage matrix is conformed by N equal columns. Matrix $\mathbf{I}$ is the square diagonal matrix containing the fault currents at nodes 1,2, …k,..N, calculated analogous to (3).

The dip matrix $\mathbf{U_{dfv}}$ contains the during fault voltages, the dips at each bus (of the N buses) due to faults at each one of the buses. The during-fault voltage at a general bus *k* when a fault occurs at that bus is contained in the diagonal of $\mathbf{U_{dfv}}$ and is zero for solid three-phase faults. Off-diagonal elements of $\mathbf{U_{dfv}}$ are the dips at a general bus *m* due to a fault at a general position *k*. For example the column *k* of $\mathbf{U_{dfv}}$ contains the retained voltage at nodes 1,2,..k,..N for a fault at node *k*, graphically presented on the one-line diagram of the power system, so called affected area. The row *m* of $\mathbf{U_{dfv}}$ contains the retained voltages at that node when faults occur at nodes 1,2,..k,..N, graphically presented on the one-line diagram of the power system, so called exposed area or area of vulnerability.

Example of the exposed area of bus PRE for three-phase faults is on Fig. 4. The exposed area for critical voltage x% means that, all faults within this area cause the drop of the voltage at bus PRE below x% of nominal voltage.

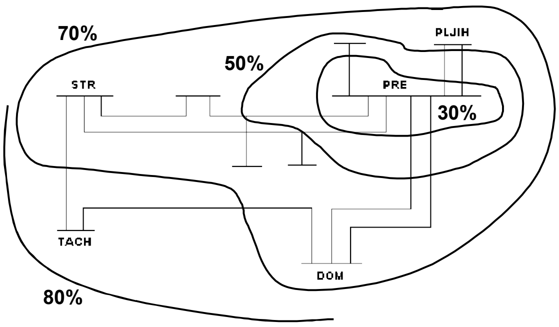


**Fig. 4** Example of exposed area [3]

For more precise description of the exposed areas is needed to take into account the faults occurring on lines. In order to simulate faults on lines additional fictitious nodes, so called fault positions, are considered along the lines (Fig. 2). The more fault positions are used to calculate the exposed areas, the more precise is the description of these areas and more accurate the stochastic assessment of dips. Unfortunately, also bigger computational effort needed to perform the calculation results from larger dip matrix. The number of rows of this larger matrix stays the same as the number of original (or physical) buses, but the number of columns increases with the number of possible fault positions (Fp).

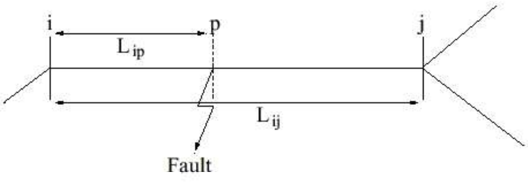


**Fig. 5** Fault position *p* on line *i-j*

The equation (4) can be used for calculation of dip matrix $\mathbf{U_{dfv}}$ extended by Fp columns corresponding to additional fault positions. Fp columns extend the impedance matrix too. The dimension of diagonal matrix **I** is then N+Fp.

For consideration of faults along lines, an additional node *p* is placed on line *i-j* (Fig. 5), the relative distance between node *i* and *p* is defined $\lambda = L_{ip}/L_{ij}$. The original $\mathbf{Z_{bus}}$ is extended by row and column *p*:

$$\mathbf{Z_{new}} = \begin{bmatrix} & & & Z_{1p} \\ & \mathbf{Z_{orig}} & & Z_{2p} \\ & & & \dots \\ & & & Z_{Np} \\ Z_{p1} & Z_{p2} & \dots\ Z_{pN} & Z_{pp} \end{bmatrix} \qquad (5)$$

The additional elements of $\mathbf{Z_{bus}}$ that correspond to the fault position *p* are calculated by using matrix elements and branch impedance of line $z_{ij}$:

$$\begin{aligned} Z_{xp} &= (1-\lambda)Z_{xi} + \lambda Z_{xj} \\ Z_{px} &= (1-\lambda)Z_{ix} + \lambda Z_{ix} \qquad x = 1,2,\dots N \qquad (6) \\ Z_{pp} &= (1-\lambda)^2 Z_{ii} + \lambda^2 Z_{jj} + 2\lambda\lambda(-\lambda)Z_{ij} + \lambda(1-\lambda)z_{ij} \end{aligned}$$

The *p* row of $\mathbf{Z_{new}}$ is not needed, if we want to know the during-fault voltages only at buses not at additional nodes (fault positions). However the element $Z_{pp}$ is needed for calculation of the fault current at fault position *p*.

Similarly other columns extend the impedance matrix. Finally, the impedance matrix has N rows and N+Fp columns.

## 5. DIP MATRICES FOR UNSYMMETRICAL FAULTS

Unsymmetrical faults can also be considered to define the exposed areas. For symmetrical faults only the positive sequence network is required to analyse the during-fault voltages. The unsymmetrical faults are modelled by symmetrical components. The system is supposed balanced allowing the independence of component sequences. The sequence impedance matrices $\mathbf{Z_{bus}^{(1)}}$, $\mathbf{Z_{bus}^{(2)}}$ and $\mathbf{Z_{bus}^{(0)}}$ are used to model the power system.

The equation (1) should be used for calculation of sequence components of during-fault voltages. Due to balanced system the pre-fault voltages contain only positive-sequence components. The equation (1) expressed by using the sequence components is:

$$\begin{aligned} \mathbf{U_f^{(1)}} &= \mathbf{U_0^{(1)}} + \mathbf{\Delta U_f^{(1)}} = \mathbf{U_0^{(1)}} - \mathbf{Z_{bus}^{(1)}} \cdot \mathbf{I^{(1)}} \\ \mathbf{U_f^{(2)}} &= \mathbf{\Delta U_f^{(2)}} = -\mathbf{Z_{bus}^{(2)}} \cdot \mathbf{I^{(2)}} \qquad (7) \\ \mathbf{U_f^{(0)}} &= \mathbf{\Delta U_f^{(0)}} = -\mathbf{Z_{bus}^{(0)}} \cdot \mathbf{I^{(0)}} \end{aligned}$$

The sequence fault currents $I_k^{(1)}, I_k^{(2)}$ and $I_k^{(0)}$ used in current vectors in equation (7) differ according to the fault type.
For a single-phase-to-ground fault:

$$I_k^{(1)} = I_k^{(2)} = I_k^{(0)} = \frac{U_{0k}}{Z_{kk}^{(1)} + Z_{kk}^{(2)} + Z_{kk}^{(0)}} \tag{8}$$

For a phase-to-phase fault:

$$I_k^{(1)} = I_k^{(2)} = \frac{U_{Ok}}{Z_{kk}^{(1)} + Z_{kk}^{(2)}}, \qquad I_k^{(0)} = 0 \tag{9}$$

For a phase-to-phase-to-ground fault:

$$\begin{aligned} I_k^{(1)} &= \frac{U_{ok}}{Z_{kk}^{(1)} + \dfrac{Z_{kk}^{(2)} \cdot Z_{kk}^{(0)}}{Z_{kk}^{(2)} + Z_{kk}^{(0)}}} \\ I_k^{(2)} &= \frac{-Z_{kk}^{(0)}}{Z_{kk}^{(2)} + Z_{kk}^{(0)}} \cdot I_k^{(1)} \\ I_k^{(0)} &= \frac{-Z_{kk}^{(2)}}{Z_{kk}^{(2)} + Z_{kk}^{(0)}} \cdot I_k^{(1)} \end{aligned} \tag{10}$$

For unsymmetrical faults the sequence dip matrices should be calculated:

$$\begin{aligned} \mathbf{U}_{\mathbf{dfv}}^{(1)} &= \mathbf{U}_{\mathbf{pref}}^{(1)} + \Delta\mathbf{U}^{(1)} \\ \mathbf{U}_{\mathbf{dfv}}^{(2)} &= \mathbf{0} + \Delta\mathbf{U}^{(2)} \\ \mathbf{U}_{\mathbf{dfv}}^{(0)} &= \mathbf{0} + \Delta\mathbf{U}^{(0)} \end{aligned} \tag{11}$$

The dip matrices for each phase voltage can be calculated:

$$\begin{aligned} \mathbf{U}_{\mathbf{dfv}}^{a} &= \mathbf{U}_{\mathbf{pref}}^{(1)} + \Delta\mathbf{U}^{(1)} + \Delta\mathbf{U}^{(2)} + \Delta\mathbf{U}^{(0)} \\ \mathbf{U}_{\mathbf{dfv}}^{b} &= a^2 \cdot \mathbf{U}_{\mathbf{pref}}^{(1)} + a^2 \cdot \Delta\mathbf{U}^{(1)} + a \cdot \Delta\mathbf{U}^{(2)} + \Delta\mathbf{U}^{(0)} \\ \mathbf{U}_{\mathbf{dfv}}^{c} &= a \cdot \mathbf{U}_{\mathbf{pref}}^{(1)} + a \cdot \Delta\mathbf{U}^{(1)} + a^2 \cdot \Delta\mathbf{U}^{(2)} + \Delta\mathbf{U}^{(0)} \end{aligned} \tag{12}$$

If we take the lowest phase voltage to characterize the dip magnitude, then each element of the dip matrix $\mathbf{U_{dfv}}$ is the lowest value of phase voltages, i.e. elements at corresponding row and column in the matrices $\mathbf{U_{dfv}^a}$, $\mathbf{U_{dfv}^b}$ and $\mathbf{U_{dfv}^c}$.

Then four dip matrices, $\mathbf{U_{dfv3p}}$, $\mathbf{U_{dfv1p}}$, $\mathbf{U_{dfv2p}}$ and $\mathbf{U_{dfv2pg}}$, characterize the bus voltages during three-phase, single-phase-to-ground, two-phase and two-phase-to-ground faults in the system respectively.

## 6. COUNTING OF VOLTAGE DIPS

The proposed modelling allows determining the magnitude of the dip for a given fault, however nothing has been said about the frequency of the event. For the stochastic assessment of dips one is interested in the expected number of events and their characteristics.

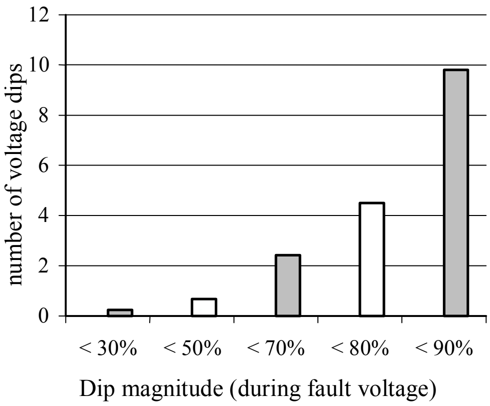


**Fig. 6** Cumulative histogram at a bus [3]

The magnitude of a dip caused by a fault at a node *k* of the system can be determined using the equations presented in section 3. How often one of those dips occurs at a given bus *m* (Fig. 3) depends on occurrence of faults at node *k*. The occurrence of faults at nodes corresponding to physical buses of the system is given by the actual fault rate of the bus. Fault positions on lines have a fault rate that is a fraction of the actual line fault rate. If more than one fault position is considered on the line the actual fault rate need to be divided by the number of fault positions.

If the fault rate of the fault position *k* is $\lambda_k$ the particular dip caused by this fault will be seen $\lambda_k$ times per year. The expected number of faults inside the exposed area is given by the sum of the fault rates of the fault positions contained in the exposed area. The expected number of dips and their characteristics can be determined by combining the dip-matrix and the fault rates corresponding to the fault positions.

Let $\boldsymbol{\lambda}$ be the vector containing the corresponding fault rate of the N buses and the Fp fault positions:

$$\boldsymbol{\lambda} = \begin{bmatrix} \lambda_1 & \cdots & \lambda_k & \cdots & \lambda_N & \cdots & \lambda_p & \cdots & \lambda_{Fp} \end{bmatrix} \tag{13}$$

The vector $\boldsymbol{\lambda}$ contains the yearly voltage drops caused by faults. However only part of these drops will be counted as dips at load buses (U < 0.9 pu). A large part of these events will result in retained voltages above 0.9 pu.

To build the cumulative histogram (Fig. 6) the during-fault voltages contained in Table I need to be grouped according to the dip magnitude of interest. Finally, in order to take into account the different fault types, the procedure described needs to be performed for each one of the during-fault voltage-matrices ($\mathbf{U_{dfv3p}}$, $\mathbf{U_{dfv1p}}$, $\mathbf{U_{dfv2p}}$ and $\mathbf{U_{dfv2pg}}$) and the resulting frequencies combined according to the probability distribution of fault types.

Table I shows how an arbitrary row of the dip matrix $\mathbf{U_{dfv}}$ corresponds to the vector $\boldsymbol{\lambda}$.

| Row *m* of dip matrix $\mathbf{U_{dfv}}$ (Dip magnitude at m) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $U_{m1}$ | ... | $U_{mk}$ | ... | $U_{mN}$ | ... | $U_{mp}$ | ... | $U_{mFp}$ |
| Vector $\boldsymbol{\lambda}$ (Frequency of event at fault positions) | | | | | | | | |
| $\lambda_1$ | ... | $\lambda_k$ | ... | $\lambda_N$ | ... | $\lambda_p$ | ... | $\lambda_{Fp}$ |

**Tab. 1** Correspondence of dip matrix and frequency of events at fault positions

## 7. CONCLUSION

Method of fault positions for the prediction of the number of voltage dips in the power system is based on fault statistics and short-circuit calculation algorithm. Described matrix-based approach using the voltage dip matrices is useful for computational implementation. If the number of voltage dips only as a function their magnitude is needed, the calculation procedure is very simple. The common short-circuit analysis software can be used, although the computation and the result interpretation is very work-intensive due to the number of fault position. For large power systems, the special voltage-dip analysis software should be used or developed.

The voltage-dip analysis software should be based on the mentioned method of fault position. By using this software the cumulative histogram of voltage dips at chosen buses can be calculated as well as other parameters describing the power quality of the system, e.g. critical distances or sensitive length of lines.

The voltage dip matrices can be also used for finding the optimal number of monitors and their emplacement in power system. On the basis of the determined monitor reach area the selection of suitable buses for location of monitors can be made to minimise the number of monitors and to get a reasonable description of the power system performance.

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## BIOGRAPHY

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