B-SPLINE FUZZY ADAPTIVE SYSTEM

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SUMMARY

In this paper, we introduce Model Reference Adaptive Fuzzy Control (MRAFC) scheme which provide controller with perfect model-tracking capability. In section 1 we review the development of fuzzy control and state out the need for adaptive fuzzy control. In section 2 we define the type of fuzzy controller, plant and reference model. In section 3 we show the derivation of the MRAFC adaptive laws. In section 4 a simulation is provide for showing the performance with a linear state-feedback controller.

Keywords: *Adaptive fuzzy control, adaptation, uncertain system, nonlinear systems, fuzzy logic*

1. INDRODUCTION

In this section, we review the development of fuzzy control and state out the fuzzy controller is a knowledge based controller that uses fuzzy set theory and fuzzy logic for knowledge representation and inference [1]. Fuzzy controllers have been successfully applied to a wide variety of applications [2, 3]. However, the problems with fuzzy controllers are that the controllers are not easy to finetune or calibrate and the evaluation of system performance is also difficult. The rule based structure of fuzzy controllers makes it difficult to mathematically characterize the closed loop system.

A „adaptive control system“ is designed so that its „adaptive controller“ has the ability to improve the performance of the closed loop system by generating command inputs to the plant and utilizing feedback information from the plant. The adaptive control scheme presented here automatically generates the fuzzy controller's knowledge base on-line as new information on how to control the plant is gathered. For instance, as shown in Figure 1, the MRAFC scheme can automatically synthesize a fuzzy controller for the plant and later tune it if there are significant disturbances or process variations.

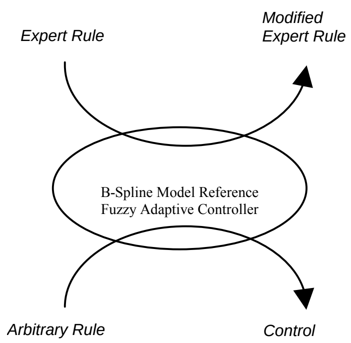

Fig. 1 MRFAC scheme for linguistic rule generation

The most important advantage of adaptive fuzzy control over conventional adaptive control is that adaptive fuzzy controllers are capable of incorporating linguistic fuzzy information from human operators, whereas conventional adaptive controllers are not. This is especially important for the systems with a high degree of uncertainty, e.g., in chemical processes and in aircraft, because although these systems are difficult to control from a control theory point of view, they are often successfully controlled by human operators.

In the following section, the construction Model Reference Adaptive Fuzzy Controller (MRAFC) is introduced. The rules of the fuzzy controller can be updated automatically in order to follow the reference model response. In designing the MRAFC, the class of fuzzy controllers, mentioned in [4], is used because of the existence of an explicit form of the controllers. Later, we will show how to apply the explicit form of the fuzzy controller to derivate a MRAC scheme which can be proved to be globally stable using the second method of Lyapunov.

2. FUZZY CONTROLLER, PLANT AND REFERENCE MODEL

In this section, we define the type of fuzzy controller used, and the structure for the plant and reference model.

2.1. Fuzzy Controller

In MRAFC scheme, the fuzzy controller, with x; y as inputs and z as output, is constructed by applying the triangular membership function, algebraic product used as logical AND operator, correlation-product inference method [1] and Center-of-Gravity(COA) method for defuzzification. The antecedent member-ship functions are triangular in shape having the properties that

$$\begin{aligned} \mu_i(x) + \mu_{i+1}(x) = 1 \quad \forall \quad \mathrm{i} \\ \mu_j(x) = 0 \quad \forall \quad \mathrm{j} \neq \mathrm{i}, \mathrm{i}+1 \end{aligned} \tag{1}$$

Therefore, for multiple inputs, there are 2^n rules activated at each time, where n is the number of inputs. In the stage of rule evaluation, the product operator is used as the logical AND operator. For defuzzification, the consequent membership functions are singleton in shape and COA defuzzification method [1] is applied. Therefore, we have

$$\mu_k = \mu_i(x)\times\mu_j(y) \tag{2}$$

$$z = \sum_{k=1}^{N}\mu_k R_k \tag{3}$$

where $N = 2^n$ is the total number of the fuzzy rules activated.

Then, as shown in [4], one can construct the generalized expression of the fuzzy controllers with multiple inputs and single output (B-Spline fuzzy system [5]). Consider a controller of the class with n inputs of $x_l \in [c_{x_l} - k_{x_l}, c_{x_l} + k_{x_l}]$ for $l = 0,1,\cdots,n$ and $u_{normal} \in [0,1]$ be the normalized output, then, the generalized expression of the class of the fuzzy controllers can be written as [4], [6]

$$u_{normal} = \sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\cdots\sum_{i_n=1}^{2} N_{i_1 i_2\cdots i_n} x_1^{i_1-1}x_2^{i_2-1}\cdots x_n^{i_n-1} \tag{4}$$

where

$$N_{i_1 i_2\cdots i_n} = \frac{1}{2^n\prod_{l=1}^{n}k_{x_l}}\left[\sum_{j_1=1}^{2}\sum_{j_2=1}^{2}\cdots\sum_{j_n=1}^{2} R_{j_1 j_2\cdots j_n}K_{j_1 j_2\cdots j_n}C_{j_1 j_2\cdots j_n}\right] \tag{5}$$

with

$$C_{j_1 j_2\cdots j_n} = \left[\frac{(-1)^{j_1}}{k_{x_1}-(-1)^{j_1}c_{x_1}}\right]^{i_1-1}\left[\frac{(-1)^{j_2}}{k_{x_2}-(-1)^{j_2}c_{x_2}}\right]^{i_2-1}\cdots\left[\frac{(-1)^{j_n}}{k_{x_n}-(-1)^{j_n}c_{x_n}}\right]^{i_n-1} \tag{6}$$

$$K_{j_1 j_2\cdots j_n} = \left[k_{x_1}-(-1)^{j_1}c_{x_1}\right]\left[k_{x_2}-(-1)^{j_2}c_{x_2}\right]\cdots\left[k_{x_n}-(-1)^{j_n}c_{x_n}\right] \tag{7}$$

2.2. Plant and Reference Model

It is assumed that the process is linear and completely controllable, and has no zeros. Moreover, assuming that all the states of the process are observable. The order of the process is denoted as n. The class of fuzzy controller is a linear state feedback controller but with an additional composite state vector x_c (see (19)) and has the form of $u = k_0 r + k_b^T x_p - k_c^T x_c$. Therefore, the fuzzy controller allows placement of the closed-loop poles at any arbitrary position. The process is described by the state equations

$$\dot{x}_p = A_p x_p + b_p u \tag{8}$$

$$y_p = c^T x_p \tag{9}$$

where A_p is assumed to be in phase-variable form

$$A_p = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_{p1} & -a_{p2} & -a_{p3} & \cdots & -a_{pn} \end{pmatrix} \tag{10}$$

$$b_p = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ b_{pn} \end{pmatrix} \tag{11}$$

$$c^T = (1,0,\ldots,0) \tag{12}$$

with $x_p = x_1$, $\dot{x}_p = x_2, \cdots, x_p^{n-1} = x_n$.

The fuzzy controller with n + 1 inputs, i.e. r and x_p can be written as

$$u_{normal} = \sum_{i_0=1}^{2}\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\cdots\sum_{i_n=1}^{2} N_{i_0 i_1 i_2\cdots i_n} r^{i_0-1}x_1^{i_1-1}x_2^{i_2-1}\ldots x_n^{i_n-1} \tag{13}$$

Then, the controller can be written as $u = G(u_{normal} - 1)$ with the gain G > 0.

Alternatively, the controller can be written as $u = \theta^T\omega$ with $\theta^T = (k_0, k_b^T, k_c^T)$ and

$$\omega^T = (r, x_p^T, x_c^T) \tag{14}$$

where x_p is the state vector with

$$x_p^T = (x_1, x_2, \cdots, x_{n-1}, x_n) \tag{15}$$

$$k_b^T = (k_1, k_2, \cdots, k_{n-1}, k_n) \tag{16}$$

with

$$\begin{aligned} k_0 &= 2GN_{211\ldots111} \\ k_1 &= 2GN_{121\ldots111} \\ &\vdots \\ k_{n-1} &= 2GN_{111\ldots121} \\ k_n &= 2GN_{111\ldots112} \end{aligned} \tag{17}$$

and x_c is the composite state vector with

$$x_c^T = (rx_1x_2\ldots x_n, rx_1x_2\ldots x_{n-1}, \cdots, x_{n-1}x_n, 1) \tag{18}$$

$$k_c^T = \left(k_{n+1}, k_{n+2}, \cdots, k_{n+n_c-1}, k_{n+n_c}\right) \quad (19)$$

with

$$\begin{aligned} k_{n+1} &= 2GN_{222\ldots222} \\ k_{n+2} &= 2GN_{222\ldots221} \\ &\vdots \\ k_{n+n_c-1} &= 2GN_{111\ldots122} \\ k_{n+n_c} &= 2GN_{211\ldots111} \end{aligned} \quad (20)$$

where $n_c = 2^{n+1} - (n+1)$ is the composite state vector with

$$\dot{x}_p = \left(A_p + b_p k_b^T\right) x_p + b_p k_0 r + b_p k_c^T x_c \quad (21)$$

$$= A_c x_p + b_c r + b_{cc} x_c \quad (22)$$

The process parameters $a_{p1}, \cdots, a_{pn}$ and b_{pn} are assumed to be unknown but constant except that the sign of b_{pn} must be known. The controller parameters $k_0, \cdots, k_n$, k_c can be adjusted by the adaptation mechanism. The reference model is identical to the process in form

$$\dot{x}_m = A_m x_m + b_m r \quad (23)$$

3. DERIVATION OF THE MRAFC ADAPTIVE RULES

In this section, we show the derivation of the MRAFC adaptive laws. For reconstruction all needed states we used auxiliary signal generator [7].

3.1. Derivation of the error equations

For the time derivative of the signal error vector $e = x_p - x_m$ the following

$$\dot{e} = \dot{x}_p - \dot{x}_m \quad (24)$$

$$= A_c x_p + b_c r + b_{cc} x_c - A_m x_m - b_m r \quad (25)$$

$$= A_m e + A x_p + b r + b_{cc} x_c \quad (26)$$

with $A = A_c - A_m$, $b = b_c - b_m$. The parameter error vector φ is defined as

$$\varphi^T = \left(b_{pn} k_0 - b_{mn}, -a_{p1} + b_{pn} k_1 + a_{m1}, \cdots\right. \quad (27)$$

$$\left., -a_{pn} + b_{pn} k_n + a_{mn}, b_{pn} k_{n+1} - 0, \cdots, b_{pn} k_{n+n_c} - 0\right) \quad (28)$$

Hence, the error equation becomes

$$\dot{e} = A_m e + b_1 \varphi^T \omega \quad (29)$$

where $b_1 = (0, \ldots, 0, 1)^T$. The system's error equation, consisting of a linear part governed by A_m and b_1, plus a nonlinear control $\varphi^T \omega$.

3.2. The Lypunov function

The choice of the Lyapunov function is normally a quadratic function of both the signal error vector e and the parameter error φ

$$V = e^T P e + \varphi^T \Gamma^{-1} \varphi \quad (30)$$

The adaptation gain matrix Γ must be positive definite and is chosen as a diagonal matrix, so Γ^{-1} is positive definite also. P must be a positive definite symmetric matrix and will follow from the adaptive law derivation shown in the following paragraph.

3.3. Differentiating V and deriving the adaptive laws

In order to obtain an asymptotically stable adaptive system, $\dot{V}$ must be negative definite. Differentiating V yields:

$$\dot{V} = e^T \left(A_m^T P + P A_m\right) e + +2 e^T P b_1 \varphi^T \omega + 2 \varphi^T \Gamma^{-1} \dot{\varphi} \quad (31)$$

By applying the second method of Lyapunov, a positive definite symmetric matrices P and Q can be found such that the first part of the equation satisfies

$$e^T \left(A_m^T P + P A_m\right) e = -e^T Q e \quad (32)$$

By putting the last two terms of the equation to zero, the adaptive laws emerges

$$2 e^T P b_1 \varphi^T \omega + 2 \varphi^T \Gamma^{-1} \dot{\varphi} = 0 \quad (33)$$

$$\dot{\varphi} = -\Gamma e^T P b_1 \omega \quad (34)$$

$$= -\Gamma \left(p^T e\right) \omega \quad (35)$$

The product $P b_1$ is a vector consisting of the n-th column p of P, and the product of this vector with the signal error vector p^T e is called the „compensated error". This compensated error is used in the adaptive laws to calculate $\dot{\varphi}$. While the model and process parameters are assumed constant, from the definition of φ it follows that

$$\dot{\theta} = -\Gamma' \left(p^T e\right) \omega = -\Gamma' e_1 \omega \quad (36)$$

with $\Gamma' = \frac{\Gamma}{b_{pn}}$. The sign of the actual adaptation gain matrix Γ' is found to depend on the sign of b_{p_n}, and so to be able to implement the adaptive law with a proper sign, the sign of b_{p_n} must be known. This condition appears in all MRAC schemes. The equation form the adaptive laws which provide a stable adaptive system. The matrix P, and so the vector p, can be calculated with Lyapunov's equation, starting with a chosen definite symmetric matrix Q.

3.4. Deriving the MRAFC adaptive laws

From the generalized expression of the class of fuzzy controller [8], we have

$$R_{j_0 j_1 j_2 \cdots j_n} = \sum_{i_0=1}^{2}\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\cdots\sum_{i_n=1}^{2} N_{i_0 i_1 i_2 \cdots i_n} D_{j_0 j_1 j_2 \cdots j_n} \quad (37)$$

with

$$D_{j_0 j_1 j_2 \cdots j_n} = \left[c_r + (-1)^{j_0} k_r\right]^{i_0-1}\left[c_{x_1} + (-1)^{j_1} k_{x_1}\right]^{i_1-1} \cdots \left[c_{x_n} + (-1)^{j_n} k_{x_n}\right]^{i_n-1} \quad (38)$$

The sensitivity of $R_{j_0 j_1 j_2 \cdots j_n}$ with respect to the $N_{i_0 i_1 i_2 \cdots i_n}$ can be obtained by taking the partial derivate and it follows that

$$\frac{\partial R_{j_0 j_1 j_2 \cdots j_n}}{\partial N_{i_0 i_1 i_2 \cdots i_n}} = D_{j_0 j_1 j_2 \cdots j_n} \quad (39)$$

Then, by using the chain rule, the adaptation law of each rule is

$$\dot{R}_{j_0 j_1 j_2 \cdots j_n} = \sum_{i_0=1}^{2}\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\cdots\sum_{i_n=1}^{2} \frac{\partial R_{j_0 j_1 j_2 \cdots j_n}}{\partial N_{i_0 i_1 i_2 \cdots i_n}} \dot{N}_{i_0 i_1 i_2 \cdots i_n} \quad (40)$$

$$= \sum_{i_0=1}^{2}\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\cdots\sum_{i_n=1}^{2} D_{j_0 j_1 j_2 \cdots j_n} \dot{N}_{i_0 i_1 i_2 \cdots i_n} \quad (41)$$

with $\dot{N}_{i_0 i_1 i_2 \cdots i_n} = \frac{\dot{k}_i}{2G}$ for $i = 0, 1, \ldots, n + n_c$. It is not dificult to show that

$$\dot{R}_{j_0 j_1 j_2 \cdots j_n} = -\gamma''\left(p^T e\right)\left(1 + a_{r_{j_0 j_1 j_2 \cdots j_n}} r\right) \times \prod_{l=1}^{n}\left(1 + a_{x_{l_{j_0 j_1 j_2 \cdots j_n}}} x_l\right) \quad (42)$$

with $\gamma'' = \frac{\gamma'}{2G}$. For each rule, the term $a_{x_{l_{j_0 i_1 i_2 \cdots j_n}}} = c_{x_1} + (-1)^{j_1} k_{x_1}$ is defined as the value of r and x_1 where the vertex of the membership function is located.

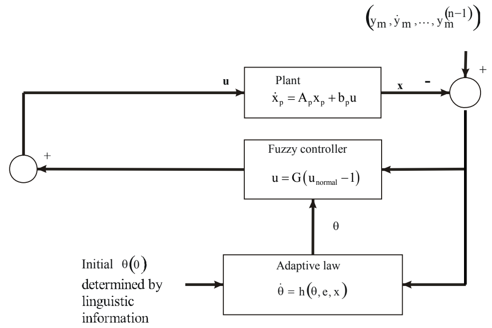

Fig. 2 Closed-loop fuzzy adaptive control system

4. EXPERIMENTAL RESULTS

The principle of the proposed control structure implementation is illustrated in Fig. 3.

The client elaborates the desired control architecture in a Simulink file and downloads it on the target station via TCP-IP protocol. The target station is equipped with the XPCTarget real-time kernel (Mathworks company), and with a data acquisition driver (Humusoft). The target station is connected to the system motor-dynamo and controls it. This strategy allows real processes teleoperations.

The DC motor is connected to amplifier card (H-bridge). The current loop was disconnected and we kept only the speed loop. Speed information is provided by generator (dynamo). The amplifier card is connected to the target station (see Fig. 3), which is equipped with DA and AD converters and encoders (Humusoft AD 512). Based on the measured output, the voltage control input is finally determined according to the algorithm (Simulink program) downloaded on the target station (PC Pentium Pro 150 MHz).

Data acquisition (evolution of the control and output signals, modification of some control parameters) is carried out in real-time using the XPC software of Mathworks company.

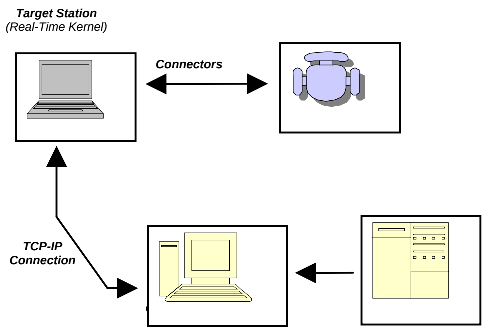

Fig. 3 Control Implementation Principle

Using the proposed adaptive control structure, the experimental responses of Fig. 4 are obtained (the sampling period is 0.001 s). From these results, it can be observed that the tracking error is close to zero (Fig. 5) which proves that good tracking performance is achieved. It can be observed that the system output follows the model output. The evolution of the fuzzy adaprive control law is illustrated in Fig. 6.

Model transfer function $W_m(s) = \dfrac{1}{s^2 + 2s + 1}$

We used as reference signal square steps between 2 - 5 V.

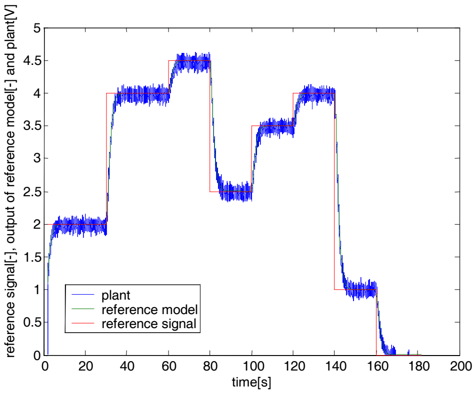

Fig. 4 Control Structure performances – reference signal, output of reference model and output of the system

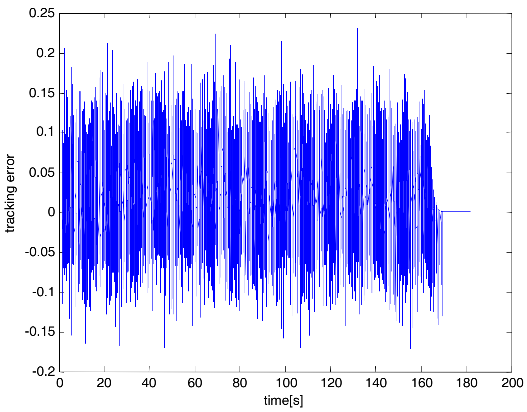

Fig. 5 Control Structure performances – tracking error

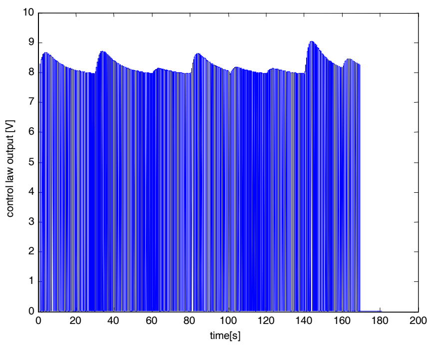

Fig. 6 Control Structure performances – evolution of the control signal

5. CONCLUSION

Most stable adaptive fuzzy control strategies published in the literature require the availability of the state vector. In addition, the system is assumed to be in canonical form. The proposed method has the advantage of being free of all these conditions. Furthermore, the adaptive fuzzy control algorithm stability is guaranteed according to the Lyapunov theory.

The global control architecture has been implemented for a motor-generator system. The experimental results show good control performance and thus the feasibility of the developed technique.

Further research is necessary to study the influence of the zero dynamics on the control performance.

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BIOGRAPHIES

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