MODEL OF THE DECISION SUPPORT SYSTEM UNDER CONDITION OF NON-DETERMINATION

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SUMMARY

Decision support systems (hereinafter DCS only), mean interactive computer systems, which assist to decision making subjects to utilize both data and models to solve non-structurized issues. These systems were established mainly based on a risk analysis, utilizing the experience/skills, conclusion making and intuition, enabling very fast and flexible analysis with a good response, enabling the application of manager intuition and judgment this way. However such decisions are often based on uncertain information, which fact requires establishment of other decision support models.

Keywords: *Decision Support System, fuzzy sets, modeling economic systems.*

1. FUNCTIONING OF DECISION SUPPORT SYSTEM

Let us suppose the manufacturing process, for which we require closed loop type of the control cycle, using a decision support system. Structure, as showed on fig. No. 1 was based on needs to optimize the controlling procedures based on external, so called limited conditions and concept targets of the top management, as well.

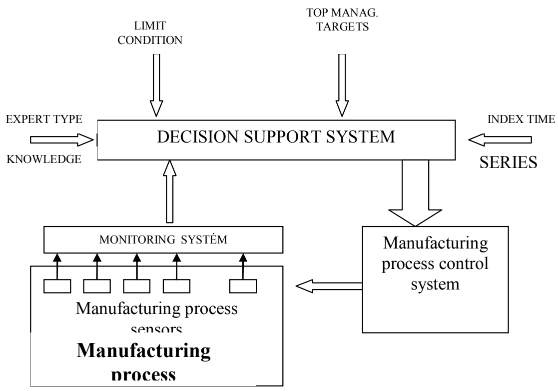

Fig. 1 General Structure of Manufacturing Process Control

Current state of the manufacturing process is indicated by means of a monitoring system. This way the decision Support System acquired immediate information covering the behavior of said manufacturing process as well as evaluation / assessment of its state. Furthermore, external limit terms/conditions, involving the economic conditions as both the main and determination conditions are entering into such Decision Support System, affecting the efficiency of said manufacturing process. Top management targets are the governing rules for behavior of the overall control system. Furthermore, we consider the fact that time series of different indices are well known, featuring the behavior of the manufacturing process. Control system is "roofed" with the Decision Support System, which suggests and selects the optimum variant of the manufacturing process.

Inherent functions of the decision Support System are shown on Fig. No. 2.

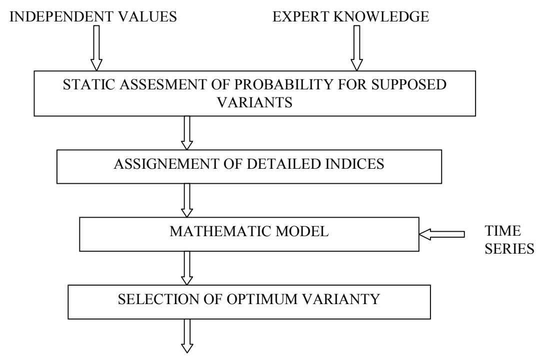

Fig. 2 Functioning of decision Support System

Independent values enter the Decision Support System, featuring either the hitherto state of appropriate manufacturing process. (via the monitoring system), either limit conditions and targets of the top management. With respect to the fact, that the combination of their occurrence has different occurrence probability, then we can assess it by means of expert knowledge. Independent values and expert knowledge enter into so called static evaluation/assessment of the probability for particular suggested variants. Vector of variants represent an output, arranged based on probability rate of occurrence for these variants. Furthermore, detailed indices are determined for respective variants, describing the above variant. Furthermore, the system of indices enters into mathematic model, which based on self-learning indices, i.e. on their time series in the past, would determine the standard for appropriate variant. Calculation is done for any and all variants, of which optimum variant of indices combination is found within the subsequent step, for subsequent manufacturing process control.

2. GLOSSARY OF TERMS

System represents particular abstraction of a real object, which is not researched by us within its complexity, but we investigate the only part of our interest, being relevant for monitoring of behavior of objects to be controlled. System itself may be described different ways and different researchers of system issues understand the system on different information and structural levels, which fact results in misunderstanding often. That is why it would not be useless for further work to implement the system description by means of epistemological level hierarchy. Particular level should be selected such a way, that the transition from the lower to the higher level would decrease the non-determination of system behavior.

Source System

At the lowest level the system is defined as a data source and that is why it is indicated as a source system, as well. It is determined by the set of quantities, time intervals and values. Particular quantities at the level of source system are understood as information sources, which within given time periods acquire any date from the set of values. None relation among particular values is available at this level. Any and all values have the same probability at the level of a source system.

Data System

In case that the source system is filled with data, either with measured or required ones, which are the values of quantities in certain time periods, then such a system is indicated to be a data system. Thus, the data system is defined as a doublet S1 = (S0, Ma), where S0 is a system definition at the level of source system, while Ma is so called matrix of activities. Each line of the above mentioned matrix is represented by a set of values, being acquired by appropriate quantity during the experiment. Knowledge of these values would enable us to estimated particular probability rates, which would reduce the indetermination of system description.

Generative Systems

Time invariant relations among system quantities is a target for transition from data system to the generative one, so that we should be able to generate the same data (under the same conditions) as involved in the Ma matrix of activities of the data system. Generative system does not consist of any data it involves relations only, which generate the data. Relations may be presented e.g. in the form determined probabilities.

Structural System

Different kinds of probability are presented in the definition of a generative system only. Perceiving of causal relationships among the quantities represents a target for transition between generative and structural levels, as well as specification of system structure and formalization of qualitative properties of appropriate relationships. After the implementation of epistemological levels into the system the issues from the point of view of system theory may be split into two disjunction sets – analysis and synthesis. Issue connected with transformation of system description from the lower to the higher epistemological level is indicated as a system analysis. Thus, the system analysis involves such issues, when we are seeking for system properties at the lower level having the knowledge of system representation at the higher level, respectively. Then the system synthesis involves such issues, when we are seeking for system properties at the higher level having the knowledge of system presentation at the lower level, respectively. Diagnostics, simulation, and other similar issues belong to the sphere of analysis, while compilation of hypotheses, planning and proposal belong to the sphere of synthesis, respectively.

Decision Support System (DCS) is understood to be a set of mechanisms (not of technical ones only) to ensured optimum control. Decision Support System is determined by a real object, for control of which it had been established. Generally we may consider it to be an information processing system, which is split out both horizontally and vertically. Horizontal segmentation at the same time respects such purposeful abstractions, which are relevant at given processing level. Vertical segmentation perceives the issue of information processing since its originating within the utilization of its processing results. Particular layers of vertical segmentation are showed on the Fig. No.3.

Decision making layer
Layer
Analytic layer
Monitoring systems layer
Information source layer

Fig. 3 DCS vertical segmentation

The highest "Decision Making Layer" involves the activities for selection of optimized controlling interventions and their application upon controlling of given system. All information on system state, pertinently trends of its development, and also knowledge on rightfulness features, controlling the system behavior, i.e. the description at the level of structurized system. This description must be available at the beginning of DCS activities already, with respect to the fact, that even during the DCS work it may be continuously improved.. Information on system state is a product of lower DCS layers.

Information source layer represents a real object in the form of a data system. Significance of said layer is covered with a fact, that this layer is the only source of information., that is why it is necessary for efficient DCS activities, that information layer involves overall (real available) information on behavior of given object, both a set of monitored information carriers (all relevant quantities), and quality of particular carriers (both for precision and time).

Basic handling with information as involved in particular information carriers of the information source layer is its acquisition (monitoring), which resides in transformation of the information into certain data structure. Furthermore, let us expect, that the above mentioned operations will be executed by means of so called monitoring systems, i.e. thru technical means for measuring, transfer, transmission and storage of data. This layer provides monitored data in a form, appropriate for next processing at the level of higher layers.

Generally, monitored data need to be processed furthermore. The data may suffer different errors, or mainly, in numerous cases it is impossible to measure required quantities directly because of the technical state of the art. But such quantities might be measured indirectly, enabling us to determine required ones. This monitoring process is not trivial generally, and in many cases affects the DCS quality considerably. Monitored data, being produced by the layer of monitoring systems are further processed on the levels of both analytic layer and synthesis one. Result involves the information on system state, based on the synthesis of analyzed monitoring data, pertinently based on repeated synthesis connected with simulation, while in the most complicated cases with the result of multilayer synthesis and simulation.

DCS utilization is befitting in such applications, where considered controlled system is complicated enough, that either the automatic operation of the monitoring systems itself and subsequent evaluation of monitored data are unreal on given state-of -art level, either complete knowledge are not available for generating the measures to be taken for controlling the considered system. Co-operation of appropriate experts is necessary in this case both during the process of identification of the state of controlled system, or in generating and selection the variant of controlling intervention. On the other hand, the DCS, via its technical, program and knowledge means creates a tool for complicated systems in case of absence of this the system controlling is inconceivable.

3. MATHEMATIC MODEL OF MAKING STRATEGIC DECISIONS UNDER NON-DETERMINATION

Upon making strategic decisions on development of certain sphere of manufacturing process, it is very advantageous to utilize the information from previous development, because there are encoded dependencies of their particular components in it. This information may be used for predicting the development for various indices and their subsequent optimization advantageously. Subject matter concerns the tasks of modeling the dependencies of various indices and subsequent evaluation of variants from the point of view of certain optimization criteria.

One of the most frequently used methods for modeling economic systems utilizes for description of development the quantities during monitored period of the functional relationship, ensuing from behaving of an independent quantity and dependent ones, as well., This must follow necessarily different, mostly considerable deflections in size of independent quantities. This trend is then transferred automatically, quite unnecessarily even into the prognosis of behavior for given quantity. Effort on the utmost approximation of time series, featuring the behavior of given quantities, is the most frequent cause of this state.

Target of this part is to propose the elimination method for most of mostly accidental deflections within the behavior of independent quantities and to establish this way a model, which would simulate the main trends of behavior of quantities more faithfully.

Let us expect, that n quantities $x_1, ..x_n$ were given, and each of them is described by the time series $x_i=\{x_{it} : t\in T\}$ and furthermore, is given quantity y, dependent on $x_1,...,x_n$, featured with a time series $y=\{y_t : t\in T\}$, as well.

Our aim is to determine the algorithm (of linear type), which would determine the quantity y, based on general values $x_1,...,x_n$.

$$(x_1,...,x_n)\rightarrow y,$$

And this in compliance with the course of time series $x_1, ...,x_n,y$ during the period T.

For this purpose, we will first make the qualitative distribution of the universe for each independent quantity x_i focused to describe the zones within the above universes, which are featured with qualitative different influences to behavior of the dependent quantity y. Theory of fuzzy sets will serve us for this purpose the best.

Thus, for each variable x_i we will define fuzzy relation $R_{ik} \tilde{\subset} \mathrm{Re}^2, k=1,...,m_i$ in the set of real numbers Re, describing values of variable x_i with approximately the same influence to behavior of quantity y. with respect to expected smooth course of functions R_{ik} we will assume, that R_{ik} is the Cartesian product of some fuzzy set $A_{ik} \tilde{\subset} \mathrm{Re}$, tj.

$$R_{ik}(x,x') = A_{ik}(x) \wedge A_{ik}(x').$$

From the point of view of inherent interpretation of this relation, we will still assume that two values x,x' of independent variable x_i have approximately the same influence to behavior of y, if exists $k, 1 \le k \le m_i$ \5k, 1 \), so that

$$x,x' \in Supp(R_{ik}) = \left\{ (z,z') \in \mathrm{Re}^2 : R_{ik}(z,z') > 0 \right\}$$

We require that the fuzzy relations R_{ik}, pertinently fuzzy sets A_{ik} will meet the following axioms:

(1) For each $i, 1 \le i \le n$, each $t \in T$ and each $x_{it} \in X_i$ exists $k, 1 \le k \le m_i$ so that $A_{ik}(x_{it}) > 0$.
(2) If $R_{ik}(x,x')>0$, then variation of the quantity y caused by the variation of quantity x_i from the value x to x' is **„small"**.
(3) Degree of probability proposition (2) depends positively on the value of proposition $R_{ik}(x,x')$.

Above mentioned axioms (excluding (1)) are formulated inaccurately and predominantly express the intuitive significance of the fuzzy relations R_{ik}.
Determination of variation in quantity y upon variation of quantity x_i from value x to x', is the basic problem, when only values of discrete time series X_i, Y are available. For the purpose of more precise formulation of the above mentioned axioms we will consider classical model of dependence x_i and y, acquired e.g. thru method of least squares approximation, i.e.

$$y = \sum_{i=1}^{n} a_i x_i + a_0 \qquad (1)$$

by means of time series x_i, y. At this point we used to allow a mistake, as we mentioned in the introduction. With respect to the fact, that relation (1) **we do not use for prediction**, but the for the analysis of dependence y in x_i only for given time period T and furthermore, to determine the fuzzy sets A_{ik}, the subsequent use of which is very robust and within a principal influence to the result, the result of utilization the relation (1) is not of so big importance, like for classical use.. By means of this deliberation we can formulate the axioms (2), (3) more precisely, i.e. thru following way.
If two values x,x' of the quantity x_i lies inside the fuzzy relation R_{ik}, , i.e..$R_{ik}(x,x') = 1$ (i.e. having really analogical influence to behavior of y), then we will require, that

$$|d(x,x')| = |y(x_1,...,x,...,x_n) - y(x_1,...,x',...,x_n)| < \varepsilon$$

Must be valid for any and all $x_1,...,x_n$ and given $\varepsilon > 0$. However, if x,x' are of approximately the same influence, i.e. $0< R_{ik}(x,x') <1$, then we admit, that value ε se may increase by certain percentage: the bigger, the smaller the value $R_{ik}(x,x')$ is, i.e. we can write

$$|d(x,x')| < \varepsilon + (1 - R_{ik}(x,x')).\varepsilon.$$

With respect to the fact, that for determination of $d_i(x,x')$ we used to use the relation (1), than is valid that $|d_i(x,x')| = |a_i(x,x')|$, and this way the axioms (2) and (3) we can convert into a next unifying one as follows:
(2') Exists such $\varepsilon > 0$, that for each i and values of $x_1,...,x_{i-1}, x_{i+1}...,x_n$,

$x,x' \in Re$ meets the condition

$R_{ik}(x,x')>0$ is valid

$$|x - x'| < \frac{\varepsilon}{|a_i|}.(2 - A_{ik}(x') \wedge A_{ik}(x)).$$

Axioms (1), (2') provide us now with a good presumption for construction of the fuzzy sets A_{ik}. To determine A_{ik} we have to specify as follows:

(a) course of the function A_{ik} ,
(b) position of the function A_{ik} v Re.

Let us suppose, that $Supp(A_{ik})=(b_1,b_2)$, then as per the axiom (2') must be valid as follows

$$|b_1 - b_2| < \frac{\varepsilon}{|a_i|}(2 - R_{ik}(b_1,b_2)) \le 2.\frac{\varepsilon}{|a_i|}$$

And similarly, if $Ker\, A_{ik} = (c_1,c_2)$, it must be valid

$$|c_1 - c_2| < \frac{\varepsilon}{|a_i|}(2 - R_{ik}(c_1,c_2)) = \frac{\varepsilon}{|a_i|}.$$

If we will suppose, that A_{ik} is of symmetrical type, then we are allowed to define course of A_{ik} as follows

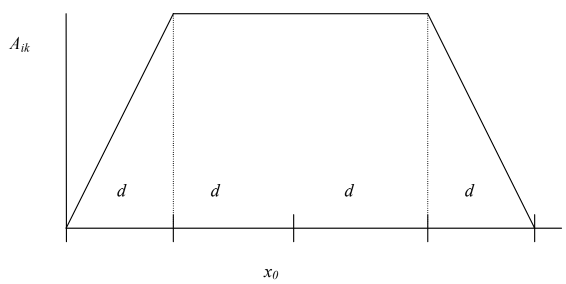

Fig. 4 Course of the fuzzy set A_{ik}

where $d = 2\frac{\varepsilon}{|a_i|}$.

then following theorem is valid (without a proof):

Fuzzy set A_{ik} defined as per the above mentioned way meets the axiom (2') for each x_0.

To resolve the task (b) we will focus to the analysis of time series X_i with respect to the scatter of length 2c = length $Supp\, A_{ik}$ and this way, except of others we can obtain even number m_i.

Thus, let say, that S is c-scatter in X_i, if exist such $x,x',\in X_i$, that is valid

1) $S=[x,x']\cap X_i$
2) $|x-x'|<2c$

For two scatters S, S' we can write $S\leq S'$ iff, for each is valid $x\in S$, $x'\in S'$, $x\leq x'$.

Then evidently exist the only system of c-scatters $\{S_i\}$ such a way, that

$$S_1 < S_2 < \ldots < S_{mi}, \bigcup S_k = X_i$$

Construction of the S_k is evident:

$$S_1 = \{x \in X_i : x - x_{\min} < 2c\}$$

where x_{min} is the least element in X_i. If are given already $S_1,\ldots,S_k$, then

$$S_{k+1} = \{x \in X_i : x \geq x_k, x - x_k < 2c\},$$

where x_k is the least element in X_i, bigger, if compared with all elements in S_k

This procedure is repeated, until $\bigcup S_k \neq X_i$. Then we select the kernel of c-scatter S_k to be an element of x_0 from the construction of fuzzy set A_{ik}, i.e.

$$x_0 = \sum_{x\in S_k} x / Card.S_k.$$

Another step is predication of behavior determination for resulting quantity y upon different qualitative inputs of particular variables x_i. Since each variable x_i has total of m_i kinds of qualitatively different values, we can obtain total of $m_1,\ldots,m_n$ relations, expressing any and all possible combinations. It is possible, that from practical point of view some combinations are unreal, however we can not exclude them upon this general consideration..

For any and each possible combination of fuzzy sets $(A_{1k1},\ldots,A_{mkm})$, identified by the vector $k=[k_1,\ldots,k_m]$, where $1\leq k_i\leq m_i$, it is necessary to determine the coefficients in following application.

If $k=[k_1,\ldots,k_m]$, then

$$y(x,k) = \sum_{i=1}^{m} a_{k,i}.x_i + a_{k,0}$$

where $a_{k,i}$ are some coefficients, whereas the criterion would be, that relation (2) is the most closest just for these values of time series $X_1,\ldots,X_m$ a Y, which are described as much as apposite by a qualitative characteristics k, i.e. for these values $x_1,\ldots,x_m$ than is the value of expression $A_{1k1}(x_1)\wedge\ldots A_{mkm}(x_m)$ the utmost of all other possible options of qualitative characteristics k.

Let us to have qualitative characteristics $k=[k_1,\ldots,k_m]$. For each time interval $t\in T$ we will determine the weight ω_t via the relation

$$\omega_t = A_{1k1}(x_{1t}) \wedge \ldots \wedge A_{mkm}(x_{mt}).$$

Then coefficients from the relation (2) would be determined such a way, that

$$\frac{\sum_{t\in T}\left[y_t - (\sum_{i=1}^{m} a_{k,i}\times_{i,t} + a_{k,0})\right]^2 .\omega_t}{\sum_{t\in T}\omega_t} \xrightarrow{!} \min$$

Other words, we consider arisen errors for these time intervals $t\in T$ as much as possible, for which the values of quantities $x_1,\ldots,x_m$ correspond to the characteristics of k as much as possible.

Classical procedure may be used for inherent determination of coefficients $a_{k,i}$, i.e. coefficients represent a solution of linear equation system with a matrix

$$\left\|\begin{array}{cccc|c} \sum\omega_t & \sum\omega_t x_{1t} & \ldots & \sum\omega_t x_{mt} & \sum\omega_t y_t \\ \sum\omega_t x_{1t} & \sum\omega_t x_{1t}^2 & \ldots & \sum\omega_1 x_{1t} x_{mt} & \sum\omega_t y_t x_{1t} \\ \vdots & & & & \vdots \\ \sum\omega_t x_{mt} & \sum\omega_t x_{1t} x_{mt} & \ldots & \sum\omega_t x_{mt}^2 & \sum\omega_t y_t x_{mt} \end{array}\right\|$$

This way we will obtain for each qualitative characteristics k a description of functional dependence $y(x,k)$, which describes behavior y much accurately, depending on $x_1,\ldots,x_m$.

Subsequent step covers the determination the functional dependence $y=y(x)$ by means of system of implications $k=[k_1,\ldots,k_m]\Rightarrow y(x,k)$. For each qualitative characteristics k and value vector x we will consider

$$[k,x] := A_{1k1}(x_1) \wedge \ldots \wedge A_{mkm}(x_m)$$

Then the determination of the value $y=y(x)$ we will acquire as follows:

First, we should split the set of indices $J=\{1,\ldots,m\}$ in dependence on vector x, into three disjunctive subsets as follows:

$$J_1 = \left\{ j \in J : x_j \in \bigcup_{t=1}^{mj} SuppA_{j,t} =: S_j \right\}$$

$$J_2 = \left\{ j \in J : x_j \in \left[x_{j,\min}, x_{j,\max}\right] - S_j \right\}$$

$$J_3 = \left\{ j \in J : x_j \notin \left[x_{j,\min}, x_{j,\max}\right] \right\}$$

where quantities $x_{j,\min}, x_{j,\max}$ are defined following way:

$$x_{j,\min} = \min(\bigcup_{t=1}^{mj} SupA_{j,t})$$

$$x_{j,\max} = \max(\bigcup_{t=1}^{mj} SuppA_{j,t}).$$

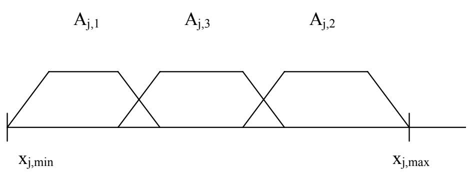

Fig. 5 Distribution of fuzzy sets

For each index $j \in J$ we will define a tetrad of values k_j, p_j, w_j, v_j, where $1 \le k_j, p_j \le m_j, v_j, w_1 \in \mathrm{Re}$, following this procedure:

1. $j \in J_1$

Then exists such index k_j, that $x_j \in Supp\ A_{j,kj}$. We will consider $P_j = k_j$, $v_j = w_j = x_j$.

2. $j \in J_2$

Then two fuzzy sets $A_{j,kj}$, $A_{j,pj}$, exist, the carriers are as close to the value x_j. as possible. Among v_j, $_j$ values we would select the utmost, pertinently the least element in kernels of these fuzzy sets.

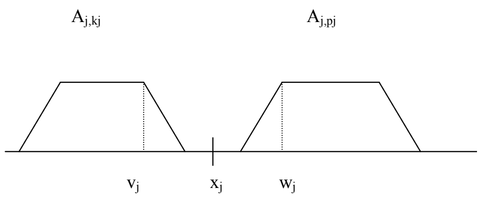

Fig. 6 Distribution of fuzzy sets for $j \in J_2$

3. $j \in J_3$

Then two fuzzy sets $A_{j,kj}$, $A_{j,pj}$ exist, as well, the carriers of which are the closest to the value x_j. The utmost elements in kernels of these fuzzy sets are selected for v_j, w_j values.

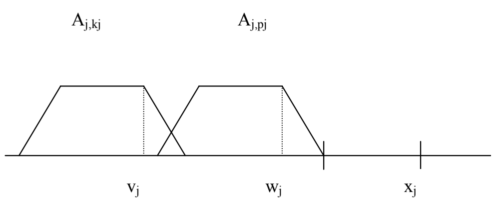

Fig. 7 Distribution of fuzzy sets for $j \in J_3$

Then the qualitative characteristics $k = \left[k_1,...,k_m\right]$, pertinently $p = \left[p_1,...,p_m\right]$ describes the value x as accurately as possible if compared all available descriptions and values of $v = \left[v_1,...,v_m\right]$, pertinently $w = \left[w_1,...,w_m\right]$ correspond to these qualitative characteristics the best. That is why for creating the value $y=y(x)$ is natural to use values $y(v,k)$ and $y(w,p)$, just together with weights, being determined by "distance" of vector x from w and v.

Let us have

$$h_1 = \sum_{j \in J_1} \left|x_j\right| + \sum_{j \in J_2} (w_j - x_j) + \sum_{j \in J_3} (x_j + w_j)$$

$$h_2 = \sum_{j \in J_2} (x_j - v_j) + \sum_{j \in J_3} (x_j - v_j).$$

Furthermore, let for given x the $K(x)$ represents next system of qualitative characteristics doublets

$$K(x) = \left\{(r,s) \in K^2 : r_1 = k_i, s_i = p_i, i \in J_2 \cup J_3\right\}$$

where K represents a set of all qualitative characteristics. Then, let us consider

$$y(x) = \frac{\sum_{(r,s) \in K(x)} (y(v,r).\left[r,v\right].h_1 + y(w,s).\left[s,w\right].h_2)}{\sum_{(r,s) \in K(x)} (\left[r,v\right].h_1 + \left[s,w\right].h_2)}$$

This way, based on knowledge of time series X and Y we will determine for specified values x a resulting quantity y. The above mentioned system may serve, except of others, as a base for generating of different development alternatives for certain indices, whereas certain probability of its existence may be allocated to each of generated variants. How to select the optimum variant under these conditions, nevertheless, remains an important question.

Thus, let us suppose, that each variant $v \in V$ is evaluated by following vector V

$$V = (v_1,...,v_m, p_v)$$

where v_i are values of appropriate resulting variables,

and p_v is a probability of variant $v \in V$.

Now, for vectors V we will define an arrangement relation as follows.

Let $J = \{1,....,m\}$ and let $\left\{J_1,...,J_r\right\}$ are disjunction distributions of set J, i.e.

$$\bigcup_i J_i = J, J_i \cap J_j = \varnothing \text{ for each } 1 \le i, j \le r.$$

Set J_i we will interpret to be preference classes for particular quantities v_k. That is why all such quantities v_k, that $k \in J_i$ are of bigger significance if compared with such arbitrary quantity v_s that $s \in J_j$, where $j > i$. Quantities, the indices of which belong to the same group J_i are of the same significance.

In next step is a **weight** h_i allocated to the group J_i, where $h_i \in \langle 0,1 \rangle$, which expresses the fact, hoe much is the group J_i of bigger importance, if compared with other groups. Roughly we can tell that as a part of total importance of vector V the group of indices J_i importance is of h_i.*100%.* evidently, it must be valid that $\sum_i h_i = 1$.

For each index $i \in J$ we will mark with a symbol q_i the following value

$$q_i = \begin{cases} +1, \text{if the higher value } v_i \text{ is advantageous} \\ -1, \text{otherwise.} \end{cases}$$

Furthermore, let symbols P_i, Q_i bear following significance:

$$P_i = \begin{cases} \max\{v_i : v \in V\}, if.q_i > 0, \\ \min\{v_i : v \in V\}, if.q_i < 0, \end{cases}$$

$$Q_i = \begin{cases} \min\{v_i : v \in V\}, if.q_i > 0, \\ \max\{v_i : v \in V\}, if.q_i < 0. \end{cases}$$

Then let us consider

$$d(v,w) = \sum_{i=1}^{r} h_i \left(\sum_{j \in J_i} q_j \cdot \frac{v_j - w_j}{w_j} \right)$$

where $v,\ w\ \in V$.

This results as follows

$$d(v,w) = \sum_{i=1}^{r} h_i \left(\sum_{j \in J_i} q_j \cdot \frac{v_j}{w_j} \right) - \sum_{i=1}^{r} h_i \left(\sum q_j \right) \le \sum_{i=1}^{r} h_i \left(\sum_{j \in J_i} q_j \cdot \frac{P_j}{Q_j} \right) - \sum_{i=1}^{r} h_i \left(\sum q_j \right) = K$$

Now, let us implement such two fuzzy linguistic variables χ_1, χ_2 that

$$\chi_1 = \langle U_1 = (0,1), \tau_1, M_1 \rangle$$
$$\chi_2 = \langle U_2 = (0,K), \tau_2, M_2 \rangle$$

where U is universe of these variables, τ is a set of terms and M is semantics. Let us consider

τ_1*={small, very, and, not, big}=*τ_2

And semantics are to be defined as follows:

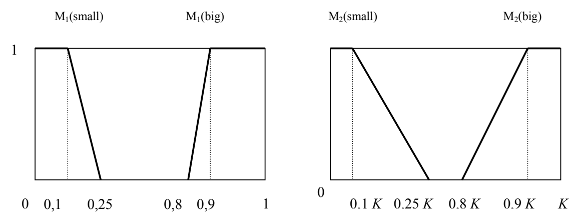

Fig. 8 Distribution of semantics *M*

Definition of other values is of classical type, i.e.

M_i*(very X)(a)=[*M_i*(X)(a)]*^{2}
M_i*(not X)(a)=1-*M_i*(X)(a)*
M_i*(X and Y)(b)=min(*M_i*(X)(b),*M_i*(Y)(b)).*

Furthermore let following rules are given:

$$X \in \chi_1, Y \in \chi_2$$

$R_1 \equiv X$ *= big* $\Rightarrow$ *Y= very - very big*
$R_2 \equiv X$ *=not very big and not very small* $\Rightarrow$ *Y= very big*
$R_3 \equiv X$ *= not small and not big* $\Rightarrow$ *Y not small*
$R_4 \equiv X$ *= small* $\Rightarrow$ *Y not very small*
$R_5 \equiv X$ *= very small* $\Rightarrow$ *Y not very-very small*

Each of these fuzzy rules R_i then represents a fuzzy relation in universe $U_1 \times U_2, R_i \tilde{\subset} U_1 \times U_2$.

Now, let us have two variants $v,\ w\ \in V$ and let us define our own relation $\le$ as follows. We will consider

$$x = P_w - P_v, y = d(v,w)$$

Let us to distinguish following cases.

I. $x \ge 0, y \ge 0.$
Then we will determine the value
$$\alpha(x.y) = \bigvee_{i=1}^{5} R_i(x,y)$$
If $\alpha(x,y) \ge \alpha_0$, we will consider $v \ge w$ (where α_0 is a level of significance).

II. $x \le 0, y \ge 0$.
Then $v \ge w$

III. $x \le 0, y \le 0$.
Then we will determine the value
$$\alpha(-x,-y) = \bigvee_{i=1}^{5} R_i(-x,-y)$$
If $\alpha(-x,-y) \ge \alpha_0$, we will consider $v \le w$.

IV. $x \ge 0, y \le 0.$
Then we will consider $w \ge v.$
Unless takes place $w \ge v$ or $w \le v$, we will consider $w \| v$.

The above mentioned procedure may be showed as following example:

Let variants $V = \{v, w\}$ are evaluated by vectors involving the following components:

1st component = **profit**
2nd component = **period of capital investment returnability**
3rd component = **number of employees**

And let in detail is valid

$$\mathbf{V} = (300, 10, 100, 0.7),$$
$$\mathbf{W} = (250, 8, 150, 0.82).$$

Indices $J = \{1,2,3\}$ will be split down into two groups as follows

$$J_1 = \{1\}, J_2 = \{2,3\}$$
$$h_1 = 0.6, h_2 = 0.4$$

From the point of view of particular vector components from V it is sure that

$$q_1 = 1, q_2 = -1, q_3 = -1$$

Thus we receive following values

I	1	2	3
Q	1	-1	-1
h_i	0.6	0.4	
Q	250	10	150
P	300	8	100

Then is valid that

$$K = 0.6(1.\frac{300}{250}) + 0.4((-1).\frac{8}{10} + (-1).\frac{100}{150}) - -0.6(1) - 0.4(-2) = 0.34$$

That is why the appropriate fuzzy sets for χ_2 are as follows

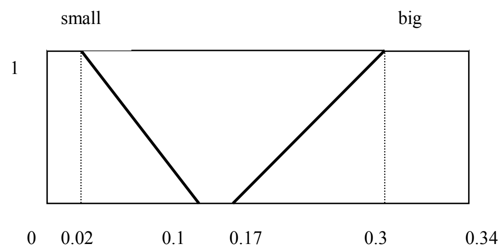

Fig. 9 Distribution of fuzzy sets for example

$$x = p_W - p_V = 0.12$$

$$y = d(v,w) = 0.6(1.\frac{300-250}{250}) + 0.4((-1).\frac{10-8}{8} + +(-1).\frac{100-150}{150}) = 0.15 \geq 0$$

Then we will determine $\alpha(x,y)$:

I	$X_{(x)}$	$Y_{(y)}$	$R_{(x,y)}$
1	0		0
2	0.2	0	0
3	0.1	1	0.1
4	0.9	1	0.9
5	0.8		0.8

That is why $\alpha(x, y) = 0.9 \geq \alpha_0 0.8$. That is why $v \geq w$.

4. MAJORANT SYSTEM STATES AND THEIR PROBABILITIES

Numerous systems exist within the sphere of manufacturing technologies, the behavior of which is not predictable under all circumstances / conditions, being under the influence of numerous both internal and external factors and relations among these factors. However, analyzing the fittability of these systems we very often need to determine, what the most frequent state are, into which the above mentioned system may fall. Since this analysis may not be done in more complicated cases exactly, results in the form of probabilities of the most frequent state are often sufficient.

Purpose of this part is to establish a system, which should enable to determine probability evaluation of occurrence the most frequent states of given system, and in the same moment, to determine such most frequent states, as well.

Thus, let us to suppose that given system φ is identified by the elements $X_1,\ldots,X_m$, where the relations among these elements are not exactly known exactly, pertinently they may not be quantified exactly. However, certain key states, into which these elements may fall. This way may be qualified also the behavior for these elements, the values of which are continuous from certain interval such a way, that this interval should be split into considerable subintervals. Thus, let each element X_i may occur in any of states $S_{i1},\ldots S_{in_i}$.

Here we must emphasize, that among these elements are to be understood also quantities from the surroundings, affecting given system.

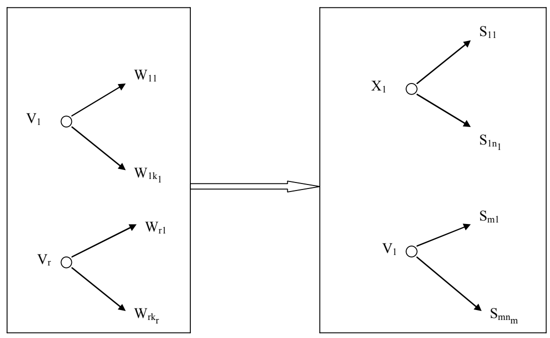

Fig. 10 States of elements and their mutual affecting

Subsequently, we will suppose, that given system has relatively big inertia, i.e. subject matter does not regard to a dynamic system featured with continuous transiting function. E.g. economic system may become typical representative of such system, pertinently a system describing the reliability in behaving of some dynamic system.

Purpose of created module is to determine, which of state vectors

$$\vec{\varphi} = (S_{1i_1}, \ldots, S_{mi_n}), \text{ where}$$
$$1 \le i_1 \le n_1, \ldots, 1 \le i_m \le m_n,$$

will occur the most probably during a time interval Δt. This is typical task of prediction the state of economic system or reliability of a dynamic system.

Let a priori probability p_{ij} is given for each state S_{ij}, independent on other external impacts/influences. These a priori probabilities are determined based on an expert estimation and/or based on other statistic methods. It is only expected that for these probabilities is valid

$$\sum_{j=1}^{m_i} P_{ij} = 1$$

From the point of view of expert evaluation, it is necessary to determine subsequently a matrix of cross-influences for appropriate system states, i.e. matrix

$$V = \left\| v_{jl}^{ik} \right\|, \ i, j, k, l \ \text{ where}$$

v_{jl}^{ik} = value, determining the influence resulting from occurrence of l-th state of the element k to probability of occurrence j-th state of variables i, $i,k,=1,\ldots,m,\ 1 \le j \le m_i,\ 1 \le k \le m_l$.

Furthermore, we will suppose that $v_{jl}^{ik} \in \{-3,-2,-1,0,1,2,3\}$, where the interpretation of these values is as follows:

Value	Significance
-3	Markedly lowers the probability
-2	Lowers the probability
-1	Moderately lowers the probability
0	Does not have influence to probability
1	Moderately increases the probability
2	Increases the probability
3	Markedly increases the probability

The inherent simulation algorithm starts with fact, that one of states for some of elements will take place in simulation, i.e. its probability would be equal to 1. Next part of said algorithm lies in determination of the influence of cross-interaction matrix to residual states. These reductions of a priori probabilities are performed by means of following algorithm.

Above all, if p_i is a probability of a priori type, then relative probability of phenomenon occurrence is

$$r_i = \frac{p_i}{1 - p_i}$$

That is why the new relative frequency $\overline{r_i}$ is determined based on following relation

$$\overline{r_i} = r_i . c_i$$

Where $c_i = \left| v_{jl}^{ik} \right| + 1$, if $v_{jl}^{ik} \ge 0$, otherwise

$$c_i = \frac{1}{\left| v_{jl}^{ik} \right| + 1}$$

Thus we receive for new probability $\overline{p}_i$

$$\overline{p}_i = \frac{\frac{p_i}{1-p_i} \cdot c_i}{1 + \frac{p_i}{1-p_i} \cdot c_i} = \frac{p_i \cdot c_i}{1 - p_i + p_i \cdot c_i}$$

Relations between probabilities p_i and $\overline{p}_i$ may be depicted graphically as follows.

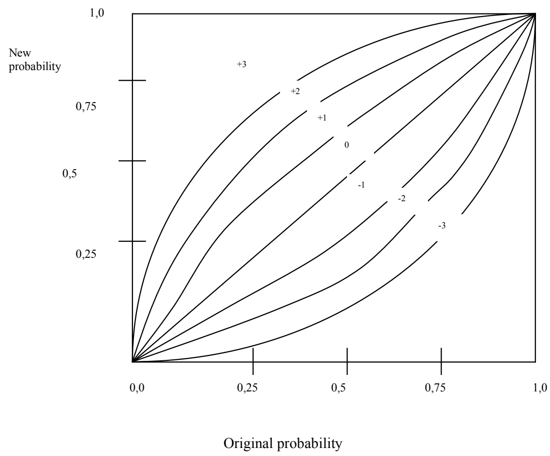

Fig. 11 Relations among probabilities

Analogical calculation is done for matrix $\overline{V} = \left\| \overline{v}_{jl}^{ik} \right\|$ where $\overline{v}_{jl}^{ij}$ is a size of influence to failing the occurrence of l-th state of element k to the probability of occurrence of j-th state of element i. Overall algorithm may be depicted as follows.

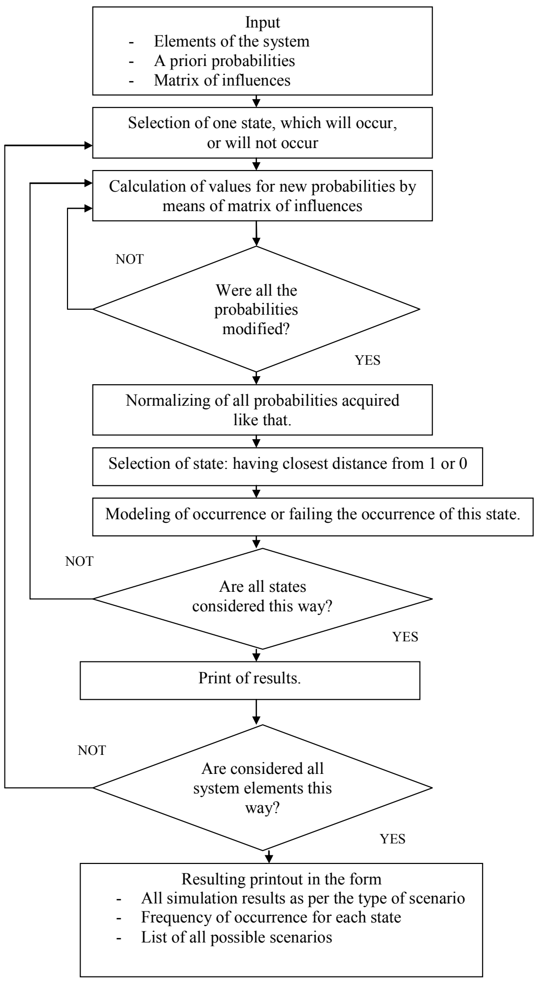

Fig. 12 Flow chart of simulation algorithm

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BIOGRAPHY

Cyril Klimeš graduated (Ing.) in 1976 at the Faculty of electrical engineering at Brno university of technology. He got his CSc. degree in 1985 at VŠB-Technical university of Ostrava and in 1991 he defended his habilitation at the same university. Now he works at the department of Informatics and computers of the Faculty of Science at University of Ostrava where he is department manager. He worked also in OKD and he has managed computer company OASA Computers, s.r.o. He is author and co-author of tens of publications and co-author of 3 patents. His scientific research is focusing on computers architecture, operation systems, information system development and e-learning.