## **ELECTRIC FIELD REGULATION AT THE CABLE ACCESSORIES USING ONE NEW NUMERICAL APPROACH**

# \* Nebojša RAIČEVIĆ, \*\*Slavoljub ALEKSI<sup>Ć</sup> \*

Department of Electromagnetics, Faculty of Electronic Engineering,

University of Niš, Aleksandra Medvedeva 14, 18 000 Niš, tel. +381 18 529447, E-mail: nraicko@elfak.ni.ac.yu \*\*\*<br>Department of Electromagnetics, Faculty of Electronic Engineering,

University of Niš, Aleksandra Medvedeva 14, 18 000 Niš, tel. +381 18 529430, E-mail: as@elfak.ni.ac.yu

#### **SUMMARY**

*The problem of electric field grading in cable components, having both theoretical and technological implications, can be framed in more general topic of field control in HV equipment. A solution may be obtained by two possible approaches: using geometric field control [11, 13, 14, 15, 17, 18], in which the field distribution depends on the arrangement of main and auxiliary electrodes or using resistive-capacitive (RC) field control [2, 4, 6, 10, 12 ], where the field distribution relies mainly on the electrical characteristics of stress grading materials.* 

*The above considerations are applicable to a wide class of MV and HV electrical devices such as insulators, bushings, spacers, voltage deviders, cable accessories (joints and terminations), etc. In the following, paper attention will be focused on the cable terminations and joints.* 

*At the places of cable connections and endings exterior cover is removed, and the radial character of electric field is disturbed. Because of high voltage, the inhomogeneous electric field exists on those parts of the cable, having the highest intensity at the ends of the covers, or screen. Cable joints and terminations represent the weakest part of a HV cable power line because of the electric field enhancement at the edge of the truncated conductors and dielectrics.* 

*The results for electric field and potential distribution at the coaxial cable terminations and joints, having exponential or ellipsoidal form, obtained by the Equivalent electrodes method (EEM), are presented in this paper. The EEM and Finite elements method (FEM) are compared. Equivalent electrodes (EE) are appointed on the end of coaxial cable, where the edge effect exists. At the great distance from terminations and joints, inside the cable, it may be considered that the field is approximately homogeneous and the charge distribution is continuous. At the cable splice, it is possible to solve the problem (electric field and electric potential distribution) as superposition of two components: the first one originates from continuous distribution of the electrical charge, and the second one from equivalent electrodes.* 

*Keywords: Equivalent electrodes method, Toroidal electrode, Edge effect, Electric field distribution, Cable terminations and joints, Equipotential surfaces.* 

## **1. INDRODUCTION**

Security of the electric power systems strongly depends on cable networks reliability. From the other side, the reliability of the cable structures depends on the way of jointing and terminating the cables, which includes the problem of cable joints [2, 5, 13, 14, 17] and terminations [1, 3, 4, 9, 11, 12, 13, 15, 18].

There is a large number of parameters that make influence on the way of the producing cable terminations, and naturally, the biggest attention is paid to the electrical field forming. It is possible to efficiently reduce electric field intensity [9, 11, 14, 15, 17, 18], especially the axial electric field component at the insulator surface. When the high voltage exists at these places, the strong and inhomogeneous electric field is formed.

Numerical evaluation of the electric field in a cable termination is carried out by adopting a model based on the electro-quasi-static approximation of Maxwell equations [1, 10].

The small size of cable terminations and joints with respect to the characteristic wave length of electromagnetic field and the low contribution of the energy associated to the magnetic field, compared to that stored in the electric field, allow the adoption of this approximation of Maxwell equations.

There are many approaches for solving the problem of minimizing electrical field intensity at the places of power cable splicing:

- 1. geometrical (by using deflectors) ;
- 2. linear resistive;
- 3. non-linear resistive field grading coatings;
- 4. refractive field grading coatings;
- 5. capacitive method; and
- 6. complex method .

The most frequently used method for minimizing electric field density in the vicinity of the sharp end that is part of the insulator screen, is based on the application of the funnel-like, appropriately modeled screen extension. This way of electric field shaping is known as geometrical modeling.

#### **2. CABLE TERMINATION**

There is a big number of cable terminations developed in last few years. Cable failures still happen, causing great economic losses, mainly because of a cable termination breakdown. For that reason any improvement in the cable termination construction is of great interest.



**Fig. 1** Non-modeled Cable termination.

Far away from the cable termination (Fig. 1) charge distribution is continuous on its conductors.

There is distributed positive charge on the interior and negative charge on the exterior conductor. Charge density per unit surface is constant in the distant regions from cable breaks, which are at the interior conductor (having radius *a* ), and at the exterior conductor (having radius *b* ), respectively

$$
\eta_a = \frac{q'}{2\pi a} \quad \text{and} \quad \eta_b = -\frac{q'}{2\pi b} \,. \tag{1}
$$

If it is presumed that such charge distribution is also in the surroundings of the cable break, the approximate expression for potential is

$$
\frac{\int_{0}^{\pi} \ln \left( \frac{L_{2} - z + \sqrt{B^{2} + (L_{2} - z)^{2}}}{L_{1} - z + \sqrt{A^{2} + (L_{1} - z)^{2}}} \right) d\theta', \qquad z \le L_{1} \le L_{2}
$$
\n
$$
\frac{\int_{0}^{\pi} \ln \left( \frac{(L_{2} - z + \sqrt{B^{2} + (L_{2} - z)^{2}})}{L_{2} - L_{1} + \sqrt{B^{2} + (L_{2} - z)^{2}}} \right) \left( z - L_{1} + \sqrt{A^{2} + (z - L_{1})^{2}} \right)}{A^{2}} d\theta', \qquad L_{1} \le z \le L_{2}
$$
\n
$$
\frac{\int_{0}^{\pi} \ln \left( \frac{L_{2} - z + \sqrt{B^{2} + (L_{2} - z)^{2}}}{L_{1} - L_{2} + \sqrt{B^{2} + (L_{1} - z)^{2}}} \right) \left( z - L_{2} + \sqrt{B^{2} + (z - L_{2})^{2}} \right)}{L_{1} - L_{2} + \sqrt{B^{2} + (z - L_{2})^{2}}} d\theta', \qquad L_{2} \le z \le L_{1}
$$
\n
$$
\int_{0}^{\pi} \ln \left( \frac{z - L_{1} + \sqrt{A^{2} + (z - L_{1})^{2}}}{z - L_{2} + \sqrt{B^{2} + (z - L_{2})^{2}}} \right) d\theta' + 2\pi I(r), \qquad L_{1} \le L_{2} \le z
$$
\n(2)

where 
$$
r, \theta
$$
 and z are cylindrical coordinates,  

$$
A^2 = r^2 + a^2 - 2ar \cos \theta';
$$
 (3)

$$
B2 = r2 + b2 - 2br \cos \theta';
$$
 (4)

$$
I(r) = \begin{cases} \n\ln \frac{b}{a}, & 0 \le r \le a \\ \n\ln \frac{b}{c}, & a \le r \le b \\ \n0, & r \ge b \n\end{cases} \tag{5}
$$

On the basis of the expression for potential, and for  $L_1 = L_2 = L$ , approximate expressions for electric field's radial and axial components are determined. If

$$
E_0 = \frac{U}{a},\tag{6}
$$

$$
K\left(\frac{\pi}{2}, m\right) = \int_{0}^{\frac{\pi}{2}} \frac{d\alpha}{\sqrt{1 - m\sin^2\alpha}}\tag{7}
$$

is the complete elliptic integral of the first kind, *m* is its squared modulus and  $U$  is the voltage that the coaxial cable is supplied by, then for electric field's radial component the followung is obtained:

$$
\frac{E_{\text{rapr}}(r,z)}{E_0} = \frac{a}{2\pi \ln \frac{b}{a}} (I_{\text{r}}(r,a,z,L) - I_{\text{r}}(r,b,z,L)),
$$
 (8)

while the axial component of the electric field is

$$
\frac{E_{\text{zapr}}(r,z)}{E_0} = \frac{a}{\pi \ln \frac{b}{a}} \left( \frac{K\left(\frac{\pi}{2}, m(r, b, z, L)\right)}{\sqrt{(r+b)^2 + (L-z)^2}} - \right), \quad (9)
$$
\n
$$
-\frac{K\left(\frac{\pi}{2}, m(r, a, z, L)\right)}{\sqrt{(r+a)^2 + (L-z)^2}} \right)
$$

where

$$
m(r,r'z,z') = k^2(r,r'z,z') = \frac{4r'r}{(r+r')^2 + (z-z')^2} \quad (10)
$$

and

$$
I_{r}(r,r',z,z') = \int_{\theta'=0}^{\pi} \frac{r - r' \cos \theta'}{\sqrt{(z'-z)^{2} + r^{2} + r'^{2} - 2rr' \cos \theta'} \left(\sqrt{(z'-z)^{2} + r^{2} + r'^{2} - 2rr' \cos \theta'} + z' - z\right)}} d\theta'.
$$
 (11)

#### **2.1. Application of the Equivalent electrodes method**

The above mentioned charge distributions do not coincide with the real ones, because the boundary conditions are not satisfied, so consequently the conductors are not of constant potential. Due to this, additional expressions are superposed to the previous ones.

If equivalent electrodes [7, 8] are used as additional elements, excellent results are obtained. Toroidal electrodes are employed as EE, having cross section radius  $a_e = L/4N$ , and with medium lines located at the places

$$
z_n = (2n - 1)\frac{l}{2},
$$
\n(12)

for  $r = a$ , e.g.,  $r = b$ , where is  $n = 1, 2, \dots, N$ .

Table 1 shows obtained convergence of the results for electric field, normalized to  $E_0$ , when the number of equivalent electrodes and the length of cable termination modeled by equivalent electrodes, are used as parameters.

$L_1 = L_2 = 3a$			$L_1 = L_2 = 6a$			$L_1 = L_2 = 9a$		
$N_1 = N_2$	$E(r = a)$	$E(r = b)$	$N_1 = N_2$	$E(r = a)$	$E(r = b)$	$N_1 = N_2$	$E(r = a)$	$E(r = b)$
	0.80257	0.53438	$\overline{2}$	0.80142	0.53235	3	0.80089	0.53041
3	0.76961	0.35885	6	0.76904	0.35872	9	0.76900	0.35859
5	0.81915	0.33648	10	0.81900	0.33619	15	0.81901	0.33611
$\overline{7}$	0.83966	0.32521	14	0.83965	0.32492	21	0.83966	0.32487
9	0.85361	0.31909	18	0.85372	0.31879	27	0.85373	0.31875
11	0.86337	0.31512	22	0.86354	0.31482	33	0.86355	0.31479
13	0.87066	0.31234	26	0.87088	0.31204	39	0.87089	0.31201
15	0.87633	0.31028	30	0.87660	0.30997	45	0.87661	0.30994
21	0.88780	0.30636	42	0.88813	0.30605	63	0.88815	0.30603
27	0.89484	0.30413	54	0.89522	0.30381	81	0.89523	0.30379
33	0.89965	0.30268	66	0.90006	0.30236	99	0.90009	0.30231
51	0.90801	0.30029	102	0.90846	0.29997	153	0.90848	0.29992
69	0.91243	0.29890	138	0.91290	0.29878	207	0.91291	0.29876
87	0.91260	0.29809	174	0.91268	0.29807	261	0.91268	0.29807

**Tab. 1** Electric field strength at the conductors, normalized to  $E_0$ , at the distance  $z = 1.5$  *a* from the cable end.

Part of the interior conductor (Fig. 2), having length  $L_3$ , is also modeled with toroidal shape.

One method for strength reduction of the electric field existing in cable ends, in the surroundings of cable breaks is to model appropriately the end of line's exterior conductor. In this way more continuous distribution of electrical potential is obtained.

In case when the exterior conductor has the same shape as broken coaxial cable's equipotential line, the best results are obtained. As these equipotentials are not mathematically explicitly defined, it is possible to use greater number of functions to describe the shape of the exterior conductor. Each of these functions has certain advantages, but also has some disadvantages.

 Using the EEM it is possible to determine potential and electric field in arbitrary chosen point of cable end region. The calculation is done for cable terminations having funnel shape, which axial sections modeled by either polynomial or exponential function (Fig. 2) and when the exterior conductor end is modeled by ellipse (Fig. 6).

EE, which replace various segments of interior conductor ends, have toroidal shapes. Their cross sections radii are determined in the following way:

- The first step is to divide the curve modeling on  $N_4$  segments and to connect these points;
- Equivalent radius  $(a_{e4})$  is determined as the forth of smallest distance among the end points;
- Centers of the equivalent electrodes are placed on the medium line ( $r = r_{p,n}$ ;  $z = z_{p,n}$ ) among the points, but on the very surface on the exterior conductor.

 If normalization of all lengths (interior and exterior conductor radius, radial and axial cylindrical coordinate and etc.) with respect to radius *a* of interior conductor is done and if equivalent electrodes' radii are

$$
a_{e1} = \frac{l_1}{4}, \ a_{e2} = \frac{l_2}{4}, \ a_{e3} = \frac{l_3}{4}, \ a_{e4n} = \frac{l_{4n}}{4}, \tag{13}
$$

and their centers are:

$$
z_{1n} = (2n-1)\frac{l_1}{2}, n = 1, 2, \dots, N_1;
$$
 (14)

$$
z_{2n} = (2n-1)\frac{l_2}{2}, n = 1, 2, ..., N_2;
$$
 (15)

$$
z_{3n} = (2n-1)\frac{l_3}{2}, n = 1, 2, ..., N_3,
$$
 (16)

then or electric scalar potential is obtained:

$$
\frac{\varphi(r,z)}{U} = \frac{\varphi_{\text{apr}}(r,z)}{U} + \sum_{n=1}^{N_1} \frac{K\left(\frac{\pi}{2}, m(r,1,z,z_{1n},a_{e1})\right)}{\sqrt{(r+1)^2 + (z-z_{1n})^2}} Q_n + \sum_{n=1}^{N_2} \frac{K\left(\frac{\pi}{2}, m(r,b,z,z_{2n},a_{e2})\right)}{\sqrt{(r+b)^2 + (z-z_{2n})^2}} Q_{N_1+n} + \sum_{n=1}^{N_3} \frac{K\left(\frac{\pi}{2}, m(r,b,z,z_{2n},a_{e2})\right)}{\sqrt{(r+b)^2 + (z-z_{2n})^2}} Q_{N_1+n} + \sum_{n=1}^{N_4} \frac{K\left(\frac{\pi}{2}, m(r,b,z,z_{2n},a_{e2})\right)}{\sqrt{(r+b)^2 + (z-z_{2n})^2}} Q_{N_1+n} + \sum_{n=1}^{N_5} \frac{K\left(\frac{\pi}{2}, m(r,b,z,z_{2n},a_{e2})\right)}{\sqrt{(r+b)^2 + (z-z_{2n})^2}} Q_{N_1+n} + \sum_{n=1}^{N_6} \frac{K\left(\frac{\pi}{2}, m(r,b,z,z_{2n},a_{e2})\right)}{\sqrt{(r+b)^2 + (z-z_{2n})^2}} Q_{N_1+n} + \sum_{n=1}^{N_7} \frac{K\left(\frac{\pi}{2}, m(r,b,z,z_{2n},a_{e2})\right)}{\sqrt{(r+b)^2 + (z-z_{2n})^2}} Q_{N_1+n} + \sum_{n=1}^{N_8} \frac{K\left(\frac{\pi}{2}, m(r,b,z,z_{2n},a_{e2})\right)}{\sqrt{(r+b)^2 + (z
$$

 $(r+1)^2 + (z+z)$  $\frac{a_1, a_{e_1}}{a_1}Q$ *n*  $N_1 + N_2 + n$  $\sum_{n=1}^{\infty} \sqrt{(r+1)^2 + (z+z_{3n})^2}$  $+\sum_{n=1}^{N_3}\frac{K(\frac{1}{2},m(r,1,z,-z_{3n},a_{e3}))}{\sqrt{N_1+\sqrt{N_2+\sqrt{N_3+\$  $+\sqrt{1^2 + (z + z_{3n})^2}$   $\sqrt{2N_1 + N_2 +$ = 1  $\frac{1}{1} \sqrt{(r+1)^2 + (z+z_{3n})^2} \xrightarrow{\mathcal{L} N_1 + N_2 + n} \frac{1}{n-1} \sqrt{(r+r_{3n})^2 + (z-z_{3n})^2}$  $\left(1-\sqrt{(r+r_{av})^2+(z-z_{av})^2}\right)^2 \approx N_1+N_2+N_3$ *n*  $f(x - z_{p}$  $N_1 + N_2 + N_3 + n$  $n=1$   $\sqrt{(r+r_{\rm nR})}$  +  $(z-z)$  $\sum_{k=1}^{N_3} \frac{K_{(2)}^2, m(r,1,2,-2,3n,ue_3)}{(r-1)^2+(r-1)^2} Q_{N_1+N_2+n} + \sum_{k=1}^{N_4} \frac{K_{(2)}^2, m(r,r_0,n,2,4n,ue_4n)}{(r-1)^2+(r-1)^2} Q_{N_1+N_2+n}$  $+r_{\text{nn}}^{\text{+}}$  +  $(z +N_2+N_3+$  $=1$   $\sqrt{(r + r_{p n})} + (z - z_{p})$ where

$$
m(r, r'z, z', r_{\text{tor}}) = k^2 (r, r'z, z') = \frac{4r'r}{(r+r)^2 + (z-z')^2 + \delta(r, r')\delta(z, z')r_{\text{tor}}^2}
$$
\n(18)

and

$$
\delta\big(x, x_n\big) = \begin{cases} 1, & x = x_n \\ 0 & x \neq x_n \end{cases} \tag{19}
$$

Relative charge is

$$
Q_n = \frac{q_n}{2\pi^2 \varepsilon a U},\qquad(20)
$$

where  $q_n$  denotes the total charge of the  $n$ -th equivalent electrode and  $\varepsilon$  is electrical permittivity of the medium.

In order to estimate the accuracy of the applied method, the end of a cut cable with  $L_3 = 0$ ,  $z_0 = 0$ ,  $b = 3a$  and  $r = a$  has been observed.

The electric field on both the interior conductor surface ( $r = a$ ) and on the inner side of the exterior conductor ( $r = b$ ) is calculated.

## **2.2. Geometrically modeled cable termination by power or exponential functions**

Applied polynomial function has following expression:

$$
r = \frac{r_0 - b}{z_0^k} z^k + b,
$$
 (21)

and the exponential

$$
r = b e^{\frac{z}{z_0} \ln \frac{r_0}{b}} = b \left( \frac{r_0}{b} \right)^{\frac{z}{z_0}}.
$$
 (22)



**Fig. 2** The end of coaxial cables having geometrically shaped termination by polynomial or exponential function.

It is considered polynomial function of the second degree  $(k = 2)$ , with the next parameters (Fig. 2):  $b = 2a$ ;  $r_0 = 3a$ ;  $z_0 = -2a$ ;  $L_1 = L_2 = 3a$ ;  $L_3 = 6a$ .

For practical applications, when cable terminations are geometrically modeled, maximum value of the axial component of electric field is mostly considered. Due to these reasons distribution of electric field's axial component is like plotted in Fig. 3.



**Fig. 3** Distribution of electric field axial component  $(E_z/E_0)$  in the radial direction  $(r/a)$ , for different axial coordinates values, modeled by polynomial function.

Axial cross-section of equipotential surfaces is presented in Fig. 4.

Distribution of electric field's intensities in the surroundings of the cable termination modeled by exponential function,  $b = 2a$ ,  $r_0 = 3a$ ,  $z_0 = -4a$ ,  $L_1 = L_2 = 3a$ ,  $L_3 = 6a$  is presented in Fig. 5.



**Fig. 4** Equipotential lines of the cable termination having exterior conductor's end modeled by polynomial function .

ISSN 1335-8243 © 2007 Faculty of Electrical Engineering and Informatics, Technical University of Košice, Slovak Republic



**Fig. 5** Electric field intensity distribution ( $E/E_0$ ) in axial direction  $(z/a)$  for different radial coordinate values for cable termination modeled by exponential function.

## **2.3. Geometrically modeled cable termination using elliptical function**

Figure 6 presents the shape of geometrically shaped cable termination with exterior conductor shaped as rotated ellipse.

The angle is determined as

$$
\alpha = \arccos\left(\frac{r_{\text{kr}} - r_0}{r_{\text{kr}} - b}\right). \tag{23}
$$

The ellipse equation, according to which is modeled the end of the coaxial cable's exterior conductor, is

$$
\frac{(z - z_{\rm c})^2}{a_{\rm po}^2} + \frac{(r - r_{\rm c})^2}{b_{\rm po}^2} = 1,
$$
 (24)

It is observed the ellipse which has:

 $b = 2a$ ,  $r_0 = 3.4a$ ,  $r_{kr} = 3a$ ,  $z_{kr} = -3a$ ,  $\alpha = 3\pi/4$ ,  $L_1 = L_2 = 2a$ ,  $L_3 = 5a$ ,  $N_1 = N_2 = 20$ ,  $N_3 = 30$ and  $N_4 = 50$ .

Distribution of electric field's axial component of the cable termination modeled by this ellipse is shown in Fig. 7.



**Fig. 6** The end of coaxial cable having geometrically shaped cable termination modeled by ellipse.



**Fig. 7** Distribution of electric field's axial component  $(E_z/E_0)$  in radial direction  $(r/a)$ , for  $z < z_0$ , for geometrically modeled cable terminations by ellipse.

 Dielectric breakdown is energetic phenomenon, so it is very important to know the density of equpotential curves.

Axial section of equipotential surfaces is presented in Fig. 8.



**Fig. 8** Equipotential lines of the cable termination with exterior conductor's end modeled by ellipse.

Table 2 shows obtained convergence of the results when the number of EE is parameter.

Electric potential and electric field components are compared with values obtained by using Finite elements method, FEM. These differences for electric potential are expressed in per cents (table 2).

N	$\varphi/U$	$E_r/E_0$	$E_z/E_0$	<b>Relative</b> error(%)
2	0.27248	$-0.068$	$-0.841$	9.17115
5	0.30219	0.208	$-1.068$	0.73303
10	0.30205	0.186	$-1.053$	0.68349
15	0.30138	0.170	$-1.040$	0.46134
20	0.30107	0.163	$-1.035$	0.35679
30	0.30077	0.157	$-1.031$	0.25710
40	0.30061	0.155	$-1.029$	0.20601
60	0.30057	0.155	$-1.029$	0.15051

**Tab. 2** Results obtained by using EEM and compared with FEM (relative error)

Relative error is maximal when only two EE are applied, and minimal for infinite number of EE.

It is possible to use analogies between planparallel and rotationally symmetric electric systems for analytical determination of electric potential and field distributions at the cable terminations [16]. Conformal mapping and Schwartz-Cristoffel's transform are applied for those calculations. For modeled cable terminations this method can not be used, but equivalent electrodes method is generally applicable.

#### **3. CABLE JOINTS**

If the distances between contact places of the interior conductors  $(z=0)$  and area where the charge distribution at the coaxial cables conductors can be assumed as uniform are different,  $L_1 \neq L_2$ , approximate expression for axial component of electrical field is

$$
\frac{E_{zsp_{apr}}}{E_0} = \frac{a}{\pi \ln \frac{b}{a}} \left( \frac{K \left( \frac{\pi}{2}, m(r, b, z, L_2) \right)}{\sqrt{(r+b)^2 + (L_2 - z)^2}} - \frac{K \left( \frac{\pi}{2}, m(r, a, z, L_1) \right)}{\sqrt{(r+a)^2 + (L_1 - z)^2}} + \right) + \frac{K \left( \frac{\pi}{2}, m(r, b, z, -L_2) \right)}{\sqrt{(r+b)^2 + (L_2 + z)^2}} - \frac{K \left( \frac{\pi}{2}, m(r, a, z, -L_1) \right)}{\sqrt{(r+a)^2 + (L_1 + z)^2}} \right) \tag{25}
$$

Toroidal electrodes are employed as equivalent electrodes (Fig. 9), having cross section radii  $a_{e1} = L_1 / 4N_1$  and  $a_{e2} = (L_2 - z_d) / 4N_2$ , and with medium lines located at the places

$$
z_{an} = (2n-1)\frac{l_1}{2}; \ r = a \tag{26}
$$

$$
z_{bn} = z_d + (2n - 1)\frac{l_2}{2}; \ r = b \ . \tag{27}
$$

## **3.3 Geometrically modeled cable joint using ellipse and power function**

There are many ways for shaping ends of cable joints of exterior conductors. The best results are obtained when shape of exterior conductor "follows" one of the equipotential surfaces, although they cannot be obtained analytically, using known mathematical functions.

In order to provide simplicity in producing of cable joints, geometrical shaping is realized by using some known mathematical functions.

The best results and the maximally reduced electric fields are obtained when cable joints are modeled with both ellipse and power function (Fig.9).

The parametric expressions of the power function used for ends modeling of the coaxial cable exterior conductors are:



**Fig. 9** Geometrically modeled cable joint using ellipse together with power function.

 $z(t) = z_d - t (z_d - z_0), r(t) = b + t^k (r_0 - b),$  (28) where  $k$  is the degree, and  $t$  is parameter having value between 0 and 1.

Radial and axial coordinates of ellipse's center are determined by using condition of "flat contact" between ellipse and power function shaped part of deflector. In these points the first derivates of functions are the same. Respecting this condition, coordinates of center of ellipse are:

$$
z_c = z_0 + \frac{a_{\text{po}}}{\sqrt{1 + \frac{1}{r'(r_0, z_0)^2} \frac{b_{\text{po}}^2}{a_{\text{po}}^2}}};
$$
(29)  

$$
r_c = r_0 - \frac{b_{\text{po}}^2}{a_{\text{po}}^2} \frac{1}{r'(r_0, z_0)} \frac{1}{\sqrt{1 + \frac{1}{(1 - \frac{b_{\text{po}}^2}{c^2})^2}}},
$$
(30)

+

po  $(1)(0, 0)$ 

 $(r_0, z_0)^2 a_{\text{po}}^2$ 

 $(z_0, z_0)^2$ 

 $r(r_0,$ 

 $r'(r_0, z)$ 

po

*a*

ISSN 1335-8243 © 2007 Faculty of Electrical Engineering and Informatics, Technical University of Košice, Slovak Republic

where

$$
r'(r_0, z_0) = -k \frac{r_0 - b}{z_d - z_0} \tag{31}
$$

is the first derivate in "contact" points.

This part of deflector is modeled by toroidal equivalent electrodes, placed in points with coordinates  $r_{pen} = r(\theta_{en})$  and  $z_{pen} = z(\theta_{en})$ , where

$$
\Theta_{\text{e}n} = \alpha_0 + (2n - 1) \frac{\alpha_{\text{pod}}}{2}; \alpha_{\text{pod}} = \frac{\alpha - \alpha_0}{N_5}.
$$
 (32)

The cable joints, modeled in this way, with:  $b = 3a$ ,  $\alpha = 3\pi/4$ ,  $L_1 = L_2 = 9a$ ,  $z_d = 8a$ ,  $r_0 = 4a$ ,  $z_0 = 5a$ ,  $a_{\text{po}} = 2a$ ,  $b_{\text{po}} = a$ , are observed.

Polynomial degree is  $k = 2$ . Calculated center of ellipse is:  $r_c = 4.6$ ;  $z_c = 6.6$ .

Distribution of the axial component of electrical field in radial direction (Fig. 10), is presented.

Equipotential curves are shown in Fig. 11.



**Fig. 10** Axial electrical field component distribution  $(E_z/E_0)$  in axial direction  $(z/a)$ , for different radial coordinate values.



**Fig. 11** Equipotential lines of geometrically modeled cable joint using ellipse together with power function, for parameters values:  $b = 3a$ ,

 $z_{d} = 8a$ ,  $r_{0} = 4a$ ,  $z_{0} = 5a$ ,  $a_{\text{po}} = 2a$ ,  $b_{\text{po}} = a$ ,  $\alpha = 3\pi/4$ ,  $L_1 = L_2 = 9a$  and  $k = 2$ .

Electric field,  $E/E_0$ , on exterior conductor surface in points  $r(\theta_{en})$ ;  $z(\theta_{en})$  for geometrically modeled cable joint, using ellipse together with power function, for parameters values:  $b = 3a$ ,

 $z_{d} = 8a$ ,  $r_{0} = 4a$ ,  $z_{0} = 5a$ ,  $a_{\text{po}} = 2a$ ,  $b_{\text{po}} = a$ ,  $\alpha = 3\pi/4$ ,  $L_1 = L_2 = 9a$  and  $k = 2$  is given with Tab. 3.

This is the best way to reduce maximal electric field in cable joints region. Maximal strength of electric field is only  $0.230722 E_0$ .



**Tab. 3** Electric field,  $E/E_0$ , on exterior conductor surface in points:  $r(\theta_{en})$ ;  $z(\theta_{en})$  for geometrically modeled cable joint using ellipse together with power function.

## **4. CONCLUSION**

The described problem solving presents very important and complex task of high power technology for producing power cables and corresponding accessories.

Finite elements method, Finite difference method, Charge simulation method, Boundary relaxation method or Boundary elements method can be applied for cable terminations and joints calculations, as well as equivalent electrodes method. The simplest calculation can be carried out using Equivalent electrodes method and it obtains very high accuracy of calculated values. Main advantages of the Equivalent electrode methods in comparison to all existing methods lie in very high precision, even in cases when relatively small number of equivalent electrodes is used. Numerical integration for calculation of some non-tabeled integrals having singular and semi-singular subintegral functions is not necessary, on the contrary to the Method of moments, what consequently causes faster calculation. In a limit case when the number of equivalent electrodes is very large (leads to infinity), results are absolutely accurate.

Equivalent electrodes method is applied to nonmodeled, as well as on some geometrically modeled cable terminations and joints, when they are assumed as deflectors of funnel form, which border lines can have elliptical shape, the shape of polynomial or exponential function, as well as the form generated as the combination of the shapes mentioned above. The results for the distribution of the electric potential, radial and axial electric field components are presented in the paper. The equipotential surfaces in the vicinity of cable terminations and joints are also presented.

ISSN 1335-8243 © 2007 Faculty of Electrical Engineering and Informatics, Technical University of Košice, Slovak Republic

#### **REFERENCES**

- [1] G. Lupo, G. Miano, V. Tucci and M. Vitelli, "Field Distribution in Cable Terminations From a Quasi-Static Approximation of the Maxwell Equations", IEEE Transactions on Dielectrics and Electrical Insulation, 3(3):399 - 409, June 1996.
- [2] B. De Vivo, G. Spagnuolo and M. Vitelli, "Variability analysis of composite materials for stress relief in cable accessories", IEEE Transactions on Magnetics, 40(2):418-425, March 2004.
- [3] F. P. Espino-Cortes, S. Jayaram, E. A. Cherney, "Stress Grading Materials for Cable Terminations Under Fast Rise Time Pulses", IEEE Transactions on Dielectrics and Electrical Insulation, Vol. 13, No. 1, pp. 430 - 435, February, 2006.
- [4] J. Rhyner, M. G. Bou-Diab, "One-dimensional Model For Nonlinear Stress Control in Cable Terminations", IEEE Transactions on Dielectrics and Electrical Insulation, Vol. 4, No. 6 , pp. 785-791, December, 1997.
- [5] D. Pommerenke, R. Jobava, R. Heinrich, "Numerical Simulation of Partial Discharge Propagation in Cable Joints Using The Finite Difference Time Domain Method", IEEE Electrical Insulation Magazine, Vol. 18, No. 6, pp. 6 - 11, November-December , 2002.
- [6] V. Tucci, J. Rhyner, "Comment on "1-Dimensional Model for Nonlinear Stress Control in Cable Terminations" [and reply]", IEEE Transactions on Dielectrics and Electrical Insulation, Vol. 6, No. 2, pp. 267 - 270, April, 1999.
- [7] D. M. Velickovic, "Equivalent Electrodes Method", Scientific Review, No. 21-22, pp. 207-248, Belgrade, 1996.
- [8] D. M. Velickovic, "Equivalent Electrodes Method Application in Fluid and Heat Flow", Facta Universitatis,  $3(15)$ , http://facta.junis.ni.ac.yu/facta/macar/macar200 303/macar200303- 01.pdf#search=%22%22Equivalent%20Electro des%20Method%22%22, pp. 951-971, Serbia, May 2003.
- [9] S. V. Nikolajevic, N. M. Pekaric-Nadj, R. M. Dimitrijevic, "Optimization of Cable Terminations", IEEE/PES Summer Meeting, SM 369-9-PWRD, Denver, Colorado, July 28 - August 1, 1996.
- [10] T. Toledo, F. Buret and P. Auriol, "Electro quasi static model for configuration using SC material: comparison of two formulations", COMPUMAG 2005, PB 5-9, Seyshan, Chine, June 25-19, 2005.
- [11] T. Toledo, F. Buret and P. Auriol, "Modeling of Cable Termination Using a Semi Conductive field Deflector by a Coupled Method", COMPUMAG 2005, PH 2-9, Seyshan, Chine, June 25-19, 2005.
- [12] Li Ming, F. Sahlen, S. Halen, G. Brosig, L. Palmqvist, "Impacts of High-Frequency Voltage on Cable-Terminations with Resistive Stressgrading", Proc. of Papers of the IEEE International Conference on Solid Dielectrics, ICSD 2004, Vol. 1, pp. 300 - 303, Toulouse, France, 05-09 July, 2004.
- [13] M. Breschi, M. Fabbri, F. Negrini, P. L. Ribani, "Combined Modeling of Cables and Joints/Terminations for The Electromagnetic Analysis of Superconducting Transactions on Applied Superconductivity, Vol. 13, No. 2, Part 2, pp. 2400 - 2403, June, 2003.
- [14] N. B. Raicevic, "Electrical Field Distribution at Cable Joints", Proc. of the Sixth International Symposium on Electric and Magnetic Fields, EMF 2003, pp. 163-166, Aachen, Germany, October 06–09, 2003.
- [15] N. B. Raicevic, "Electrical Field and Potential Distribution at the Cable Termination", Proc. of the IEEE International Conference on Electric Power Engineering POWER TECH'99, CD-ROM BPT 99- 471-30, Budapest, Hungary, August 29 – September 2, 1999.
- [16] N. Raicevic, "Electric Field Calculation at Cable Terminations Using Conformal Mapping and Equivalent Electrodes Method", Proc.of the 51st International Scientific Colloquium IWK 2006, Paper 3\_3\_7 (on CD), Ilmenau, Germany, 11-15 September, 2006.
- [17] N.B. Raicevic, "Electrical Field Modeling at the Cable Joints", Proc.of the 7th International Conference on Applied Electromagnetics PES 2005, CD ROM, pp. 19-30, Nis, Serbia, 23-25 May, 2005.
- [18] N. B. Raicevic, "Electrical Field and Potential Modeling of Cable Termination", the 8<sup>th</sup> International IGTE Symposium on Numerical Field Calculation in Electrical Engineering, pp. 21-23, Graz, Austria, 21-23 September, 1998.

## **BIOGRAPHIES**

**Nebojsa B. Raicevic** was born in Nis, Serbia, on March 27, 1965. He received the Diploma (with honors) and MSc degrees in electrical engineering from the University of Nis. He is currently a Research and Teaching Assistent in the Faculty of Electronic Engineering, Department of Electromagnetics, University of Nis. His research interests include cable terminations and joints, numerical methods for electromagnetic problems solving, nonlinear electrostatic problems and magnetic field calculation of coils and permanent magnets.

**Slavoljub R. Aleksic** was born in Bercinac, Serbia. He received B.Sc. degree from the Faculty of Electronic Eng. of Nis, Department of Telecommunications, in 1975. He received M.Sc. degree in 1979. and Ph.D. degree in 1997. He became Associate Professor in 2003. Since 2004 he is the chief of the Department of Electromagnetics at the Faculty of Electronic Engineering of Nis. The area of his research is electromagnetic field theory.