ROBUST FIXED POINT TRANSFORMATIONS BASED ADAPTIVE CONTROL OF AN ELECTROSTATIC MICROACTUATOR

József K. TAR
Institute of Intelligent Engineering Systems, John von Neumann Faculty of Informatics, Óbuda University, Bécsi út 96/B H-1034 Budapest, Hungary, tel.: +36-1-666-5543, e-mail: tar.jozsef@nik.uni-obuda.hu

ABSTRACT

In this paper a potential application of a novel adaptive controller based on robust fixed point transformations of local basin of attraction in iterative learning is suggested on the basis of numerical simulation results. Its main advantage is its simplicity in comparison with the more sophisticated adaptive approaches in nonlinear control that normally are based on the application of Lyapunov functions. While the Lyapunov function based parameter tuning normally is complicated, is restricted to special models and typically is sensitive to unknown external disturbances, the here resented method needs only approximate setting of actually only two adaptive parameters and it efficiently can compensate the effects of the external perturbations. The electrostatic micro-actuators (EμAs) seem to be typical application area for this control that needs simple and fast calculations on the basis of quite approximate available model.

Keywords: *Adaptive Control, Robust Fixed Point Transformations, Iterative Control, Nonlinear Systems' Control, Geometric Approach in Control*

1. INTRODUCTION

The present paper was inspired by the work by Vagia, Nikolakopoulos and Tzes who suggested the application of a robust switching PID controller coupled to a feed-forward compensator for controlling an electrostatic micro-actuator (EμA) in [1]. In their approach the precise non-linear model of a given EμA was linearized in certain set-points as typical operating points and the LMI technique was used in the design phase to stabilize separate PID controllers that were determined in the vicinity of these set points. Such kinds of controllers have to switch at the boundaries within which static PID parameters are set. More precisely *"Each PID controller stabilizes the convex hull formed by the current and neighboring linearized subsystems of the electrostatic micro-actuator indexed according to the operating point."* The design typically was made by minimization of quadratic cost functions. In the paper simulation results were presented for responses to step functions as controller-inputs. In many cases this approach is really more reasonable than the design of complicated nonlinear model-based controllers and has significant occurrence in the literature as e.g. in [2], [3]. In its philosophy it can be related to hierarchical solutions as e.g. the idea of the situational control of more complicated systems in which in the practice typical regimes of operation can be identified [4].

It worths noting that the effects of possible occurrence of drastic modeling errors and unknown external disturbances were not addressed in this approach. Furthermore, though the model of the (EμA) they took from [3] and [5] is nonlinear, it does not seem to be too complicated and contains only a few parameters. The switching controller they proposed contains considerable number of further parameters depending on the number of the intervals of linearization. The need for the rejection of external disturbances and compensating the effects of modeling errors in a simple way in this case naturally arises.

As is well known the most sophisticated classic model-based adaptive controllers in robotics and certain mechanical devices are the "Slotine-Li Adaptive Controllers" and the "Adaptive Inverse Dynamics Controller" [6] that utilize the linear dependence of the equations of motion on the dynamic parameters of the system to be controlled. Since these controllers assume that the system under control is not subjected to external disturbances they can compensate only the effects of the parameter errors of the initial model they apply [7]. In spite of several improvements as e.g. in [8], [9] these deficiencies remained significant. It is worth noting, too, that many factors that can be ignored in the macro-world play an important role in the micro-world that have to be taken into account in modeling and control [10]. One of such factors is friction the identification of the parameters of various models of which is a complicated task [11]. It can be noted that apart from the viscous term these model parameters are not linearly separable that is needed for the applicability of the above two methods. Consequently the adaptive control of an EμA is a typical task for which the application of robust fixed point transformations of local basin of attraction is expedient to be proved. In the sequel at first the essence of the method is provided, then the model of the EμA and its appropriateness to this control approach will be shown. Finally simulation results and concluding remarks will be provided.

2. THE EXPECTED–REALIZED RESPONSE SCHEME AND FIXED POINT TRANSFORMATIONS

Several control tasks can be formulated by using the concepts of the appropriate *"excitation" Q* of the controlled system to which it is expected to respond by some prescribed or *"desired response" r^d*. The appropriate excitation can be computed by the use of some *inverse dynamic model $Q = \varphi(r^d)$*. Since normally this inverse model is neither complete nor exact, the actual response determined by the system's dynamics, ψ, results in a *realized response r^r* that differs from the desired one: $r^r \equiv \psi(\varphi(r^d)) \equiv f(r^d) \neq r^d$. It is worth noting that the functions $\varphi()$ and $\psi()$ may contain various hidden parameters that partly correspond to the dynamic model of the system, and partly pertain to unknown external dynamic forces acting on it. Due to phenomenological reasons the controller can manipulate or "deform" the input value from r^d so that $r^r \equiv \psi(r^d_*)$. The main idea is that via the introduction of an iterative process as $r_{n+1} = \Psi(r_n; r^d)$ the solution of the problem can be found as $r_n \to r_*$. If the iteration is convergent and this convergence is fast enough the solution practically can be well approximated. For showing that for *Single Input – Single Output (SISO)* systems the appropriate deformation can be defined as some *Parametric Fixed Point Transformation* consider the iteration generated by some function as $x_{n+1} = G(x_n|x^d)$. In order to apply iterations let us consider the set of the real numbers as a linear normed space with the common addition and multiplication with real numbers, and with the absolute value $|\bullet|$ as a norm. It is well known that this space is *complete*, i.e. it is a *Banach Space* in which the *Cauchy Sequences* are convergent. Due to that, using the norm–inequality, for a convergent iterative sequence $x_n \to x_*$ it is obtained that

$$\begin{aligned} &|G(x_*) - x_*| \le |G(x_*) - x_n| + |x_n - x_*| = \\ &\quad = |G(x_*) - G(x_{n-1})| + |x_n - x_*|. \end{aligned} \tag{1}$$

It is evident from (1) that if G is continuous then the desired fixed point is found by this iteration because in the right hand side of (1) both terms converge to 0 as $x_n \to x_*$. The next question is giving the necessary or at least a *satisfactory condition of this convergence*. It also is evident that for this purpose contractivity of $G(\bullet)$, i.e. the property that $|G(a) - G(b)| \le K|a - b|$ with $0 \le K < 1$ is satisfactory since it leads to a *Cauchy Sequence* ($|x_{n+L} - x_n| \to 0$ $\forall L \in \mathtt{N}$):

$$\begin{aligned} &|x_{n+L} - x_n| = |G(x_{n+L-1}) - G(x_{n-1})| \le \ldots \\ &\quad \le K^n |x_L - x_0| \to 0 \;\; as \quad n \to \infty \end{aligned} \tag{2}$$

For the role of function $G(x; x^d)$ a novel fixed point transformation was introduced in [12] that is rather "robust" as far as the dependence of the resulting function on the behavior of $f(\bullet)$ is concerned (3). This robustness can approximately be investigated by the use of an affine approximation of $f(x)$ in the vicinity of $x_\star$ and it is the consequence of the strong nonlinear saturation of the sigmoid function $\tanh(x)$:

$$\begin{gathered} G(x|x^d) := (x + K)\times \\ \left[1 + B\tanh(A[f(x) - x^d])\right] - K \\ \mathtt{if} f(x_\star) = x^d \;\; \mathtt{then} \\ G(x_\star|x^d) = x_\star \\ G(-K|x^d) = -K, \\ G(x_\star|x^d)' = (x_\star + K)ABf'(x_\star) + 1. \end{gathered} \tag{3}$$

It is evident that the transformation defined in (3) has a proper ($x_\star$) and a false ($-K$) fixed point, but by properly manipulating the control parameters A, B, and K the good fixed point can be located within its basin of attraction, and the requirement of $|G'(x_\star|x^d)| < 1$ can be guaranteed. This means that the iteration can have considerable speed of convergence even nearby $x_\star$, and the strongly saturated tanh function can make it more robust in its vicinity, that is the properties of $f(x)$ have less influence on the behavior of G. It is not difficult to show that in the case of *Single Input – Single Output (SISO)* systems the $G(x|x^d)$ functions can realize contractive mapping around $x_\star$. Qualitatively it can be stated that a small value of the parameter A opens a wide "window" in the vicinity of the realized response, while parameter K can yield an additional shift to speed up the tuning. Practically these parameters can be set via simulations: by the use of a simple PID-type controller one can observe the order of magnitude of the desired and simulated responses, and A and K can be set accordingly. It can be noted that instead of the tanh function any sigmoidal function with the property of $\sigma(0) = 0$, e.g. $\sigma(x) := x/(1 + |x|)$ can be similarly applied, too.

It has be noted that within each control cycle only one step can be executed in the iteration. If the adaptation is faster than the dynamics of the system to be controlled appropriate result can be expected even in this case, too. This approach is similar to the application of *"Cellular Neural Networks (CNN)"* for image processing. In relation to the operation of CNNs the concept of *"Complete Stability"* can be introduced that means that a static input picture is mapped to a static output picture following a short dynamic transition of the physical state of the CNN. If the input picture is not static but varies "slowly" in comparison with the "speed" of the CNN's internal dynamics varying picture is mapped to varying output [13]. In spite of using a single step during one control cycle from each point of view the improvement may be considerable.

3. THE MODEL OF THE EμA

The EμA corresponds to a micro-capacitor whose one plate is attached to the ground while its other moving plate is floating in the air as e.g. in [14]. In the present paper the model considered was taken from [1]. Accordingly, the equation of motion of the system is given as follows

$$\ddot{q} = \frac{-b * \dot{q} - kq + \varepsilon A U^2/(2(\eta_{max} - q)^2) + Q_d}{m} \tag{4}$$

in which $b = 1.4 \times 10^{-5} kg \cdots$ is the viscous damping of the motion of the EμA in air, $k = 0.816 N/m$ is a spring constant, $A = (400 \times 10^{-6} m)^2$ denotes the area of the plate, $m = 7.096 \times 10^{-10} kg$ is its mass, $\eta_{max} = 4 \times 10^{-6} m$ is the distance between the plates when the spring is relaxed, q is the displacement of the plates from the relaxed position, $\varepsilon = 9 \times 10^{-12} C^2/(N \cdot m^2)$ is the dielectric constant, Q_d denotes the external disturbance forces, and U denotes the control voltage e.g. the physical agent by the help of which the plate's displacement can be controlled. It can be seen that (4) is singular near $q = \eta_{max}$, therefore for controllability allowable displacements of the micro-capacitors plate in the vertical axis were $q \in [0.1, 1.3] \times 10^{-6} m$ that was deemed necessary in order to guarantee the stability of the linearized open-loop system in [1]. It can be noted that though the stability of the control of the linearized system obtained significant attention in the literature in the near past (e.g. [15], [16], [17]), from our present point of view linearization is not interesting at all. In our approach the *realized response of the system* is $\ddot{q}$, therefore from purely kinematical point of view we can prescribe the *desired response* $\ddot{q}^{Des}$. It also is trivial that $\frac{\partial \ddot{q}}{\partial U} > 0$ if $U > 0$ that is the derivative of $G(x_\star|x^d)' = (x_\star + K)ABf'(x_\star) + 1$ can be made flat in (3) for guaranteeing the proper convergence of the proposed method. In the sequel simulation examples will be presented for the simple PID and the novel adaptive version of the control proposed.

4. SIMULATION EXAMPLES

To study the effect of the modeling errors in the simulations the controller assumed the approximate model parameters as follows: $\hat{A} = 0.8A$, $\hat{m} = 1.2m$, $\hat{b} = 1.2b$, $\hat{k} = 1.2k$, $\hat{\eta}_{max} = 0.8 * \eta_{max}$, and $\hat{\varepsilon} = 0.8\varepsilon$. External disturbance forces that may originate e.g. some vibration of the basis plate on which the EμA is assembled were modeled by third order spline functions. It can be noted that while the estimation techniques based on Kalman filters normally assume some restriction regarding the statistical nature of the disturbances, in our case no such restrictions are needed. To obtain comparable results with [1] (in its Fig. 8 the response for the jumping control input needed about $2ms$, the prescribed relaxation used the following PID settings: $\ddot{q}^{Des} = \ddot{q}^N + 3\Lambda^2(q^N - q) + 3\Lambda(\dot{q}^N - \dot{q}) + \Lambda^3 \int_{t_0}^{t}(q^N(\xi) - q(\xi))d\xi$ with $\Lambda = 10^4/s$ in which $q^N(t)$ denotes the nominal trajectory. Du to the very fast motion required in the simulations Euler's integrating formula was used with constant time-resolution of $\delta t = 10^{-5}s$. In the simulations with fixed time-step length the above function was used as follows: $\ddot{q}^{Req}_{n+1} = G(\ddot{q}^{Req}_n, \ddot{q}_n|\ddot{q}^{Des}_{n+1}) = (\ddot{q}^{Req}_n + K_{ctrl})(1 + B_{ctrl}\sigma(A_{ctrl}(\ddot{q}_n - \ddot{q}^{Des}_{n+1}))) - K_{ctrl}$ in which the "deformed input" is referred to as the "required" value. Representative results are given in Fig. 1.

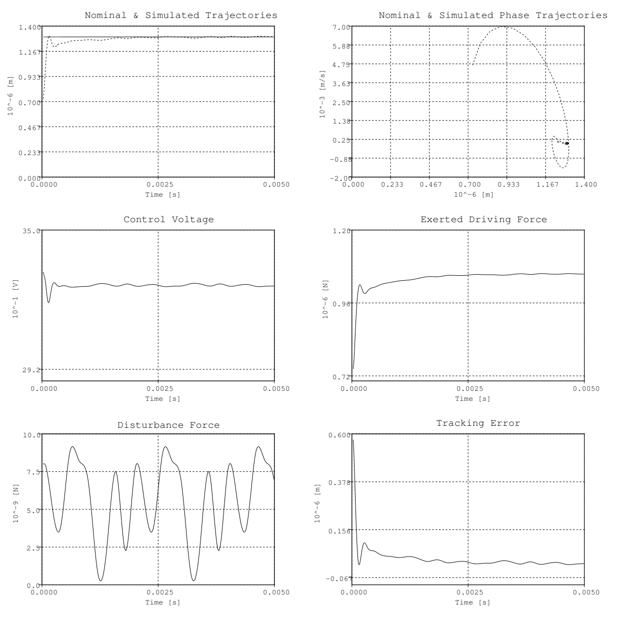

Fig. 1 The response of the non-adaptive PID controller for step input: the displacement q vs. time, the phase trajectory of the displacement, the control voltage U and the control force (according to the exact model parameters), the external disturbance forces, and the trajectory tracking error

Via observing the order of magnitude of the occurring accelerations the following *adaptive control parameters* were set: $K_{ctrl} = -10 \times 50m/s^2$, $B_{ctrl} = 1m/s^2$, and $A_{ctrl} = 1.5 \times 10^{-3}s^2/m$. The adaptive counterpart of the previously considered motion is given in Fig. 2.

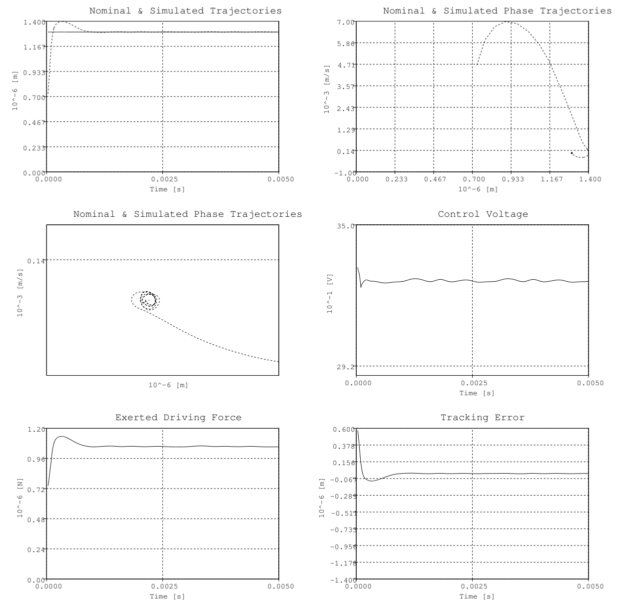

Fig. 2 The response of the adaptive PID controller for step input: the displacement q vs. time, the phase trajectory of the displacement and its zoomed details, the control voltage U and the control force (according to the exact model parameters), and the trajectory tracking error (the external disturbance forces were the same as in Fig. 1)

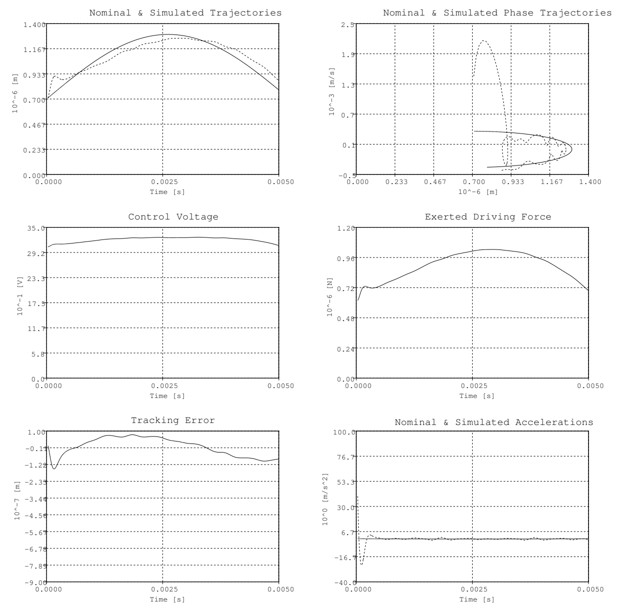

Fig. 3 The response of the non-adaptive PID controller for sinusoidal input: the displacement q vs. time, the phase trajectory of the displacement, the control voltage U and the control force (according to the exact model parameters), the trajectory tracking error, and the acceleration tracking (the external disturbance forces were the same as in Fig. 1)

The improvement by adaptivity is evident.

It is rather interesting to see the tracking properties of the controllers for continuously varying nominal trajectory, e.g. for a sinusoidal nominal motion (Fig. 3). The adaptive counterpart of the results displayed in Fig. 3 are given in Fig. 4

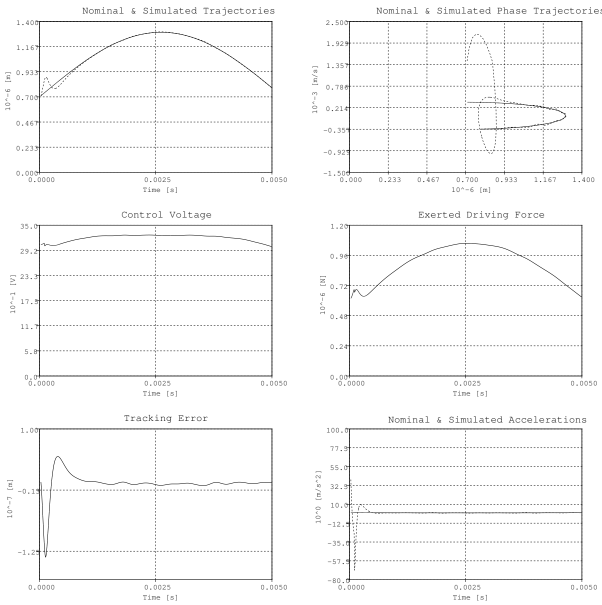

Fig. 4 The response of the adaptive PID controller for sinusoidal input: the displacement q vs. time, the phase trajectory of the displacement, the control voltage U and the control force (according to the exact model parameters), the trajectory tracking error, and the acceleration tracking (the external disturbance forces were the same as in Fig. 1)

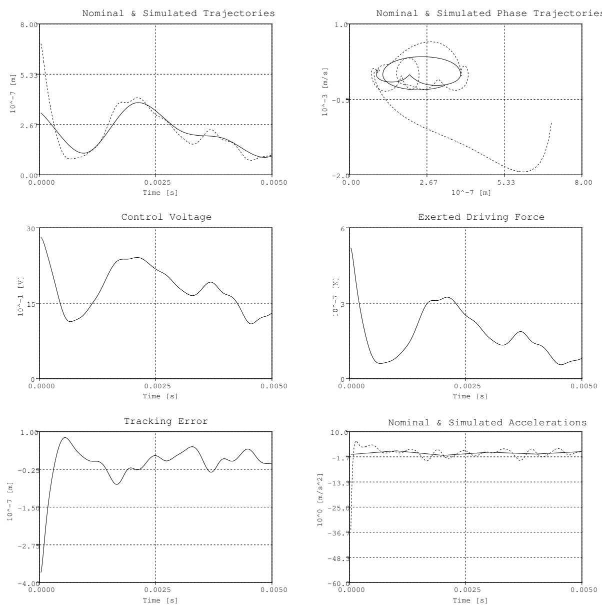

Fig. 5 The response of the non-adaptive PID controller for "irregular" input: the displacement q vs. time, the phase trajectory of the displacement, the control voltage U and the control force (according to the exact model parameters), the trajectory tracking error, and the acceleration tracking (the external disturbance forces were the same as in Fig. 1)

For investigating the operation of the controllers less "canonical" nominal trajectories third order spline functions were made to generate nominal trajectories.

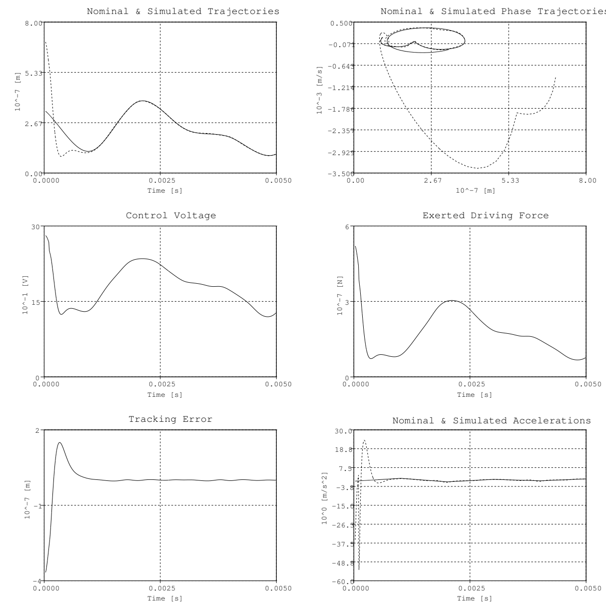

Fig. 6 The response of the adaptive PID controller for "irregular" input: the displacement q vs. time, the phase trajectory of the displacement, the control voltage U and the control force (according to the exact model parameters), the trajectory tracking error, and the acceleration tracking (the external disturbance forces were the same as in Fig. 6)

The non-adaptive controller's operation is described by Fig. 5. The adaptive counterpart of Fig. 5 is Fig. 6. The improvement by adaptivity is evident.

It is worthy of note that in spite of the disturbance forces the system smoothly approximates the acceleration of the nominal trajectory, that is a great advantage in comparison with the also simple and very witty idea of the robust "Variable Structure/Sliding Mode Controller" (e.g. [18], [19], [20]) in which the approximation of the phase trajectories may be questionable (this is the essence of the phenomenon called "chattering"). We note that the here suggested adaptive control was successfully used for softening the sharp parameters of the original VS/SM controller in [21], too.

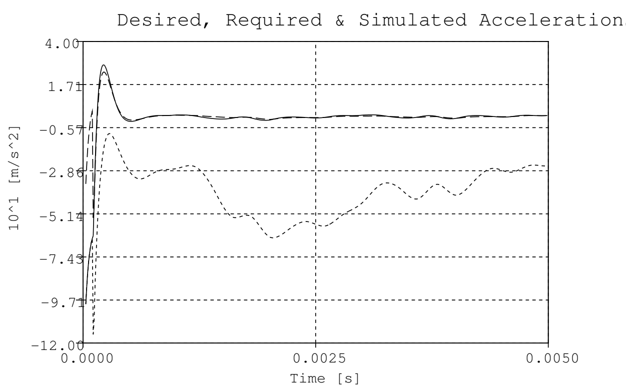

Fig. 7 The "desired" (solid line), the "required" (densely dashed line) and the "realized" (dashed line) accelerations

To better reveal the essence of the operation of the proposed controller the "desired", "required" and "realized" accelerations are described in Fig. 7. It is evident that following a short initially converging stage the "realized" values closely approximate the "desired" ones that considerably differ from the "required" acceleration. This latter difference well illustrates the deformation of the input caused by function G that is the essence of the adaptivity and the "iterative learning" feature of such controllers. In this manner the relaxation of the tracking error purely kinematically prescribed by the PID controller is precisely realized in spite of the modeling errors and the external disturbances.

5. CONCLUSIONS

In this paper a simple, robust fixed point transformation based iterative, adaptive controller was suggested for controlling a particular model of EμA. It was shown that the present approach needs far less number of parameters than the switching controllers based problem tackling in [1] in which the model is linearized around several working points, and the controller has to memorize that points and the appropriate PID parameters belonging to the appropriate intervals. Since normally the parameter B_{ctrl} can be chosen to be equal to 1, appropriate setting of only two ones, A_{ctrl} and K_{ctrl} is satisfactory, plus the setting of a single parameter for the PID controller, Λ is required. It was shown that the here presented controller can compensate the effects of quite considerable modeling errors (about 20% in each model parameter) and that of the external disturbances, too. These issues were not addressed in [1].

It must be noted that the here proposed approach needs the estimation of the acceleration of the plate while the simple PID based method and the switching controller needs information only on the position and the velocity for the feedback. Furthermore, it works with an iterative sequence of local basin of attraction, therefore in the case of extreme disturbances and parameter errors the algorithm can quit this basin of attraction. On this reason it is expedient to make preliminary simulation investigations to properly set its control parameters and to investigate the range of convergence.

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BIOGRAPHY

József K. Tar was born on 03. 12. 1957. In 1981 he graduated as physicist (MSc) at the Roland Eötvös University in Budapest, Hungary. He defended his PhD in the field of robotics, modeling and control of robots. He had the following most important jobs: 1981-1984: TUNGSRAM Co. Ltd. – Central Research Institute for Physics, Budapest; 1985-1993: various positions at TUNGSRAM Co. Ltd.;1993-: involved in education at Budapest Tech and its legal predecessor; 2009-: director of the Transportation Informatics and Telematics Knowledge Center; 2010-: involved in education at Óbuda University- He has 18 book chapters, 48 journal publications, 177 papers in conference proceedings, 4 invited lectures/tutorials.