LAWS OF INFORMATION IN DECENTRALIZED CONTROL SYSTEMS

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ABSTRACT

This paper deals with the laws of information in decentralized control systems. In the case of decentralized control of complex systems, often referred to hierarchical systems, theoretically analyze the possibility of using an n-dimensional information theory to deal with such systems. N-dimensional information theory is an interesting tool to deal with these systems. On the basis of information theory the partition laws of information rates are listed. These laws are fundamental requirements for complex systems that must be met and by their quantification one can be able to compare these complex control systems and track how different parts of the system communicate and collaborate with others, etc. In the final section the deterministic partition law of information rates and its possibility of use for dynamical systems are presented.

Keywords: *cybernetics, entropy, information, variety, probability, regulator, decentralized control*

1. INTRODUCTION

Control systems are becoming increasingly interesting from the perspective of information structure and information flow between various parts of systems. American scientist – the pioneer of cybernetics W. Ross Ashby in his book Introduction to cybernetics presented a different view on controllers and controlled systems as is now generally used. Ashby's approach is based on the theory of information, where systems are regarded as transmission channels, which are capable to transfer different amounts of information. His work is among the experts of this field, highly prized today. Based on similar considerations and simple examples he worked on in the world well known law of requisite variety [1]. Its importance for the theory of automatic control is very significant. It is therefore necessary to devote sufficient attention. For its verification it is necessary to deal with information theory, probability and entropy, since variety is very closely related to entropy. W. Ross Ashby formulated this law for centralized control systems, but it is interesting that a similar principle can be also applied to systems with hierarchical structure, for example to decentralized control systems. In this regard continued his student Roger C. Conant, and he formulated laws of information's, which control such systems [2]. His major contribution was the decomposition of systems into subsystems by using theory of information. The only one disadvantage is that these laws are not quite good applicably in the study and analysis of dynamic systems. The role of this paper is to remove this disadvantage and modify these laws for use in decentralized dynamical systems and verify its on a selected example of a complex system.

2. PRINCIPLE OF REQUISITE VARIETY

The subject of regulation is very wide in its applications, covering as it does most of the activities in physiology, sociology, ecology, economics, and much of the activities in almost every branch of science and life. Further, the types of regulator that exist are almost bewildering in their variety. Let us therefore forget all about regulation and simply suppose that we are watching two players, *R* and *D,* who are engaged in a game (see Table 1):

Table 1 Table of game for two players

		R		
		α	*β*	*γ*
D	*1*	*b*	*a*	*c*
	2	*a*	*c*	*b*
	3	*c*	*b*	*a*

D must play first, by selecting a number, and thus a particular row. *R*, knowing this number, then selects a Greek letter, and thus a particular column. The italic letter specified by the intersection of the row and column is the outcome. If it is an *a, R* wins; if not, *R* loses. Examination of the table soon shows that with this particular table *R* can win always. Whatever value *D* selects first, *R* can always select a Greek letter that will give the desired outcome. *R* has, complete control of the outcome [3].

From all possible tables let us eliminate those that make *R's* game too easy to be of interest. Let us consider, then, only those tables in which no column contains a repeated outcome. When this is so *R* must select his move on full knowledge of *D's* move; i.e. any change of *D's* move must require a change on *R's* part (see Table 2) [3].

In the case if outcome is letter *k*, then we have transformation which uniquely specifies a set of outcomes. It can now be stated that the variety in this set of outcomes cannot be less than *D's* variety/*R's* variety i.e., in this case 9/3. In general if no two elements in the same column are equal, and if a set of outcomes is selected by *R,* one from each row, and if the table has *r* rows and c columns, then the variety in the selected set of outcomes cannot be fewer than r/c.

We can now look at this game still with the restriction that no element may be repeated in a column from a slightly different point of view. If the varieties are measured logarithmically, and if the same conditions hold,

Table 2 Table of game for two players, where no column contains the same two results

		R		
		α	β	γ
D	1	f	f	k
	2	k	e	f
	3	m	k	a
	4	b	b	b
	5	c	q	c
	6	h	h	m
	7	j	d	d
	8	a	p	j
	9	l	n	h

then the theorem takes a very simple form. Let V_D be the variety of D, V_R that of R, and V_O that of the outcome (all measured logarithmically). Then the previous section has proved that V_O cannot be less, numerically, than the value of $V_D - V_R$. Thus V_O's minimum is $V_D - V_R$. If V_D is given and fixed, $V_D - V_R$ can be lessened only by a corresponding increase in V_R. Thus the variety in the outcomes, if minimal, can be decreased further only by a corresponding increase in that of R. This is the law of Requisite Variety: *only variety in R can force down the variety due to D* [4]. **Only variety can destroy variety.**

3. PROBABILITY, ENTROPY AND MUTUAL INFORMATION

Probability, entropy and mutual information or transmissions are the main part of information theory. Information theory is science investigating quantitative laws related to transfer and processing of information. At the present, the theory of information is necessary mathematical tool for the study of various control processes, what is often forgotten. Receiving, processing, transmission and storage of information of various kind is the main task of each control system. To solve different tasks of information theory it is necessary to learn how quantitative measure the content of information which is transmitted or received through the communication channel.

3.1. Probability

Let's look at the system, which is able to receive final amount of states $x_1, x_2, \ldots, x_n$ with probabilities $p_1, p_2, \ldots, p_n$, where

$$p_i = P(X \sim x_i), \tag{1}$$

is probability, that system X receive state x_i. It is clear that

$$\sum_{i=1}^{n} p_i = 1. \tag{2}$$

These data can be written in table form (see Table 3), where in the first row are states of the system and in the second row are corresponding probabilities [5].

Table 3 Probability table

x_i	x_1	x_2	$\cdots$	x_n
p_i	p_1	p_2	$\cdots$	p_n

This table is identical with table table of random discrete variable with values $x_1, x_2, \ldots, x_n$ and their probabilities $p_1, p_2, \ldots, p_n$.

3.2. Entropy

As a measure of uncertainty of the system (or discrete variable) X in information theory is special characteristic called entropy. Entropy is main concept of information theory. Entropy of the system is sum of products of probabilities of different states and the logarithms of these probabilities with the opposite sign

$$H(X) = -\sum_{i=1}^{n} p_i \log_a p_i \tag{3}$$

The main properties of entropy are: entropy is equal to zero, if one of the states is certain and others are impossible; entropy has maximum if all states of the system are equally probable. Consider the simplest system X, which has two equally probable states (see Table 4).

Table 4 Probability table of system with two equally probable states

x_i	x_1	x_2
p_i	$\dfrac{1}{2}$	$\dfrac{1}{2}$

Actually entropy calculated by using (3) is $H(X) = -\left(\dfrac{1}{2} \log_2 \dfrac{1}{2} + \dfrac{1}{2} \log_2 \dfrac{1}{2} \right) = 1$. Base unit for entropy is bit (from English word **bi**nary digi**t**) [5].

3.3. Mutual information

Mutual information, transmission or transinformation is measure of amount by which two variables are related, i.e., are not statistically independent. Transmission between two variables is denoted by $T(X_1 : X_2)$ and defined through probabilities or by

$$
\begin{aligned}
T(X_1 : X_2) &= H(X_1) + H(X_2) - H(X_1, X_2) \\
&= H(X_1) - H(X_1 \mid X_2) \\
&= H(X_2) - H(X_2 \mid X_1)
\end{aligned}
\tag{4}
$$

It is symmetric and measures the amount by which knowledge of one variable reduces uncertainty about other, or the amount by which the joint uncertainty $H(X_1, X_2)$ is smaller than it would be with X_1 and X_2

independent (then, $H(X_1, X_2) = H(X_1) + H(X_2)$). The transmission is therefore a measure of relatedness between variables, which accounts for its usefulness in systems science. $T(X_1 : X_2)$ falls in the interval $[0, \min\{H(X_1), H(X_2)\}]$, being 0 if and only if X_1 and X_2 are statistically independent and maximum if and only if one variable determines the other [2].

3.4. Method and examples for calculation of entropy

For application of the principle of requisite variety for each control systems, it is necessary to design method of measurement and calculating of entropy. Like entropy of experiments, entropy of signals is also related with probability.

Let consider urn with 20 balls. 8 balls are white, 5 balls are black and 7 balls are red. Balls are well mixed. Balls are drawing out in random way and then put back into the urn. After several attempts, we found that each colour ball is drawing out with likelihood ratio as the ratio of the frequency representation of each colour. So the probability that white ball is drawing out is 8/20=40%, the black ball is 5/20=25% and red ball is 7/20=35% (see Fig. 1).

Entropy of this experiment calculated by using (3) is $H = -0,4.\log_2 0,4 - 0,25.\log_2 0,25 - 0,35.\log_2 0,35 = 1,5589$ bit.
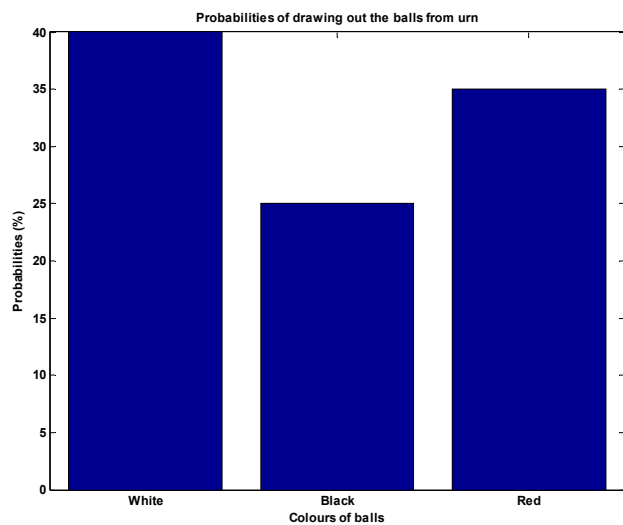

Fig. 1 Probability of drawing out the balls from urn

Consider now discrete signal with values 1, 2 and 3 during 10 seconds with sampling time 1 second, according to this sequence $S = \{1, 3, 3, 2, 1, 3, 3, 1, 2, 3\}$ (see Fig. 2).

Value 1 occurs in signal 3 times, value 2 occurs 2 times and value 3 occurs 5 times in signal. This occurs are in literature often called as multiplicity. From multiplicity is possible to calculate probabilities of values 1, 2 and 3. Probability for value 1 is 3/10, for value 2 is 2/10 and for value 3 is 5/10. Entropy for this discrete signal with three values is $H = -\dfrac{3}{10}.\log_2\dfrac{3}{10} - \dfrac{2}{10}.\log_2\dfrac{2}{10} - \dfrac{5}{10}.\log_2\dfrac{5}{10} =$

$= 1,4855$ bit. In the left part of figure (see Fig. 2) is discrete signal and in the right part is histogram. Based on this histogram we can determine the frequency of specific value, then probability of specific value and so entropy of current signal.
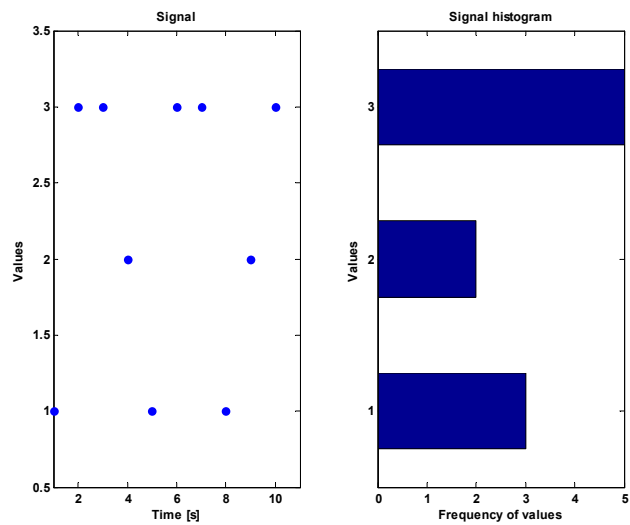

Fig. 2 Discrete signal with histogram

4. DECENTRALIZED CONTROL SYSTEMS

As complex systems we can consider systems that consist of large number of parts that are interacting with each other. Since certain difficulties arise in control such system as a whole, i.e. centralized, it is appropriate to use decomposition – particulate whole system to N independent subsystems and control whole system as decentralized.

4.1. Decomposition of complex system into subsystems

Look now at complex system in terms of information theory. Since in most cases we deal with dynamical systems – systems that change their properties depending on time and information theory has no natural relationship to real time, it is necessary to add some additional considerations. The information theory can be used only in the event that we have measured real data from complex system.

After each sustained observation of a complex dynamic system, we get some protocol, which include values of observed variables and corresponding time

Table 5 Protocol of complex system

Time	States of system
0	$(x_1^0, \ldots, x_n^0)$
1	$(x_1^1, \ldots, x_n^1)$
...	...
j	$(x_1^j, \ldots, x_n^j)$
...	...

vector. Thus, if the variables are $x_1, x_2, ..., x_n$, and the system, as n-tuple, was observed at times indicated by superscripts, the protocol will consist of an actual value given to each symbol (see Table. 5).

A most important case occurs when the system is state-determined, i.e. when n-tuple x^{j+1} is the same function of x^j whatever the value of j. The protocol can be represented equivalently by the single function $x^{j+1} = f(x^j)$, or $x' = f(x)$ [6]. When this is so, an important new set of variables, $2n$ in number, $(x_1, x_2, ..., x_n, x_1', x_2', ..., x_n')$ represents the transitions, i.e. the behaviour in real time, one state of the new system (of $2n$ variables) corresponding to one transmission of the old.

Between these new $2n$ variables all the various measures of entropy, transmission and interaction may be computed exactly as over any other set of variables, but they can now be interpreted by their relations with real time. Thus $T(x_i : x_i')$ measures something very close to concept of "cause and effect", for it measures how much the latter value at x_i is dependent on the prior value. If larger sets are studied $(x_1^0, ..., x_n^0, x_1^1, ..., x_n^1, x_1^j, ..., x_n^j)$, such a transmission as $T(x_i^0 : x_i^j)$ would measure how much the variable x_i shows j steps later in time, the effect of its earlier value. The measure thus catches something essential in the concept of x_i's "memory" [6].

4.1.1. Example of the decomposition of complex system

Consider now complex system according to [7], which can be write in the form

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = -x_1(t) - x_3(t) + u_1(t)$$
$$\dot{x}_3(t) = x_4(t) \tag{5}$$
$$\dot{x}_4(t) = -x_3(t) - x_1(t) + u_2(t)$$

Respectively in discrete form using forward difference (Euler method), we have $\dot{x}(t) = \dfrac{x(t+T) - x(t)}{T}$, then

$$x_1(k+1) = x_1(k) + Tx_2(k)$$
$$x_2(k+1) = x_2(k) - Tx_1(k) - Tx_3(k) + Tu_1(k)$$
$$x_3(k+1) = x_3(k) + Tx_4(k) \tag{6}$$
$$x_4(k+1) = x_4(k) - Tx_3(k) - Tx_1(k) + Tu_2(k)$$

where T is sampling interval equal to 0.1 seconds. After simple simulation of this system with initial conditions [0.5 0.3 -0.5 -0.1] and inputs represented with Heaviside functions, we get the protocol mentioned above with corresponding sample data, which represent the four state variables measured at corresponding sampling interval. On the following figure (see Fig. 3) is time behaviour of state variables of complex system for selected time period.
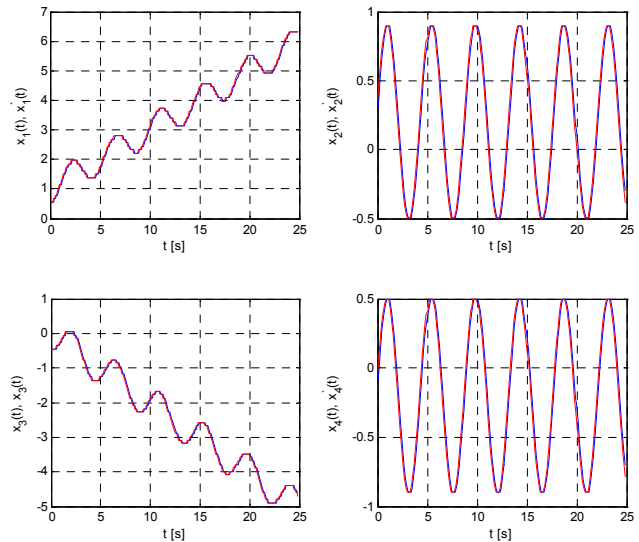

Fig. 3 Time behaviour of state variables with selected initial conditions and inputs

From these measured data we get sets of variables, generally labelled as S_j, where $S_j = \left\{ x_{j1}, x_{j2}, ..., x_{jn_j} \right\}$. Then we can denote $H(S_j)$ as entropy of $S_j = \left\{ x_{j1}, x_{j2}, ..., x_{jn_j} \right\}$. It is a nonnegative measure of the total amount of activity or variability in the set S_j, and it is calculated by a formula similar to (3). The entropy of the union of two sets S_i and S_j is denoted by $H(S_i, S_j)$. The observed transmission between S_i and S_j is denoted $T(S_i : S_j)$ and is defined as follows [8]

$$T(S_i : S_j) = H(S_i) + H(S_j) - H(S_i, S_j) \tag{7}$$

This is nonnegative measure of strength of the relation between S_i and S_j [8].

Suppose now a system is in fact "nearly decomposable" into subsystems $S_1, S_2, ..., S_m$, then it would be expect that the constraint holding between the subsystem over a short time span would be weak compared to the constraint within them. The constraint holding within the j-th subsystem $S_j = \left\{ x_{j1}, x_{j2}, ..., x_{jn_j} \right\}$ will be measured reasonably by T_{wj} defined as

$$T_{wj} = T(x_{j1} : x_{j1}' : x_{j2} : x_{j2}' : \cdots : x_{jn_j} : x_{jn_j}') \tag{8}$$

since this transmission measures nonindependance of all variables in the subsystem over time increment. The strength of the relation between the i-th and j-th subsystems over one time increment is measured by T_{bij}, defined as

$$T_{bij} = T\left(\left\langle S_i, S_i' \right\rangle : \left\langle S_j, S_j' \right\rangle \right) \tag{9}$$

and the constraint between all subsystem over one time increment is measured by T_b

$$T_b = \left(\left\langle S_1, S_1^{'} \right\rangle : \left\langle S_2, S_2^{'} \right\rangle : \cdots : \left\langle S_m, S_m^{'} \right\rangle \right) \qquad (10)$$

In a nearly decomposable system T_{bij} is small compared to $T_{wi} + T_{wj}$, for all i and j. The calculation of these transmissions thus allows verification of a proposed grouping of variables into subsystems. The last question remains – how does infer a proposed grouping from the observations? A reasonable measure of the effect of x_i on x_j one time increment later is the normalized transmission t_{ij}

$$t_{ij} = \frac{T(x_i : x_j^{'})}{H(x_j^{'})} \qquad (11)$$

According to above mentioned equations it is necessary to calculate the entropy of all state variables and joint entropy of pairs $(x_i, x_j^{'})$ where $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$. To calculate normalized transmissions t_{ij} between states of complex systems equation (11) is used and all these values are written in the table (see Table 6).

Table 6 Normalized transmission t_{ij} between states of system

t_{ij}		$x_j^{'}$			
		$x_1^{'}$	$x_2^{'}$	$x_3^{'}$	$x_4^{'}$
x_i	x_1	0,7955	0,5328	0,2573	0,1328
	x_2	0,5410	0,7416	0,1415	0,2416
	x_3	0,2629	0,1427	0,7832	0,5427
	x_4	0,1410	0,2416	0,5415	0,7416

According to table (see Table 6), which contains calculated values of t_{ij} we are able to decide whether complex system is decomposable intoto subsystems. According to values from table (see Table 6), it is clear that a complex system is nearly decomposable into subsystems S^1 and S^2, where $S^1 = \{x_1, x_2\}$ and $S^2 = \{x_3, x_4\}$. This is only the initial idea about complex system and his subsystems, which however must be tested. Based on [8], the complex system is decomposable into subsystems, if couplings between variables that make up the subsystem are stronger than the couplings between separate subsystems. In our case, this fact can be verified only by calculate the transmissions using equations (8) and (9). Then for subsystem S^1 is $T_{w_1} = T(x_1 : x_1^{'} : x_2 : x_2^{'}) = 13, 47$ bit/sample time and for S^2 is $T_{w_2} = T(x_3 : x_4^{'} : x_3 : x_4^{'}) = 13, 63$ bit/sample time. T_{w_1}

and T_{w_2} are transmissions that measures the nonindependance of all variables in the subsystem S^1 and S^2 over one time increment – sample time. It remains only to calculate the strength of coupling between subsystems S^1 and S^2 using equation (9). Then $T_{b_{12}} = T\left((S_1, S_1^{'}) : (S_2, S_2^{'}) \right) = \left((x_1^{'}, x_1, x_2, x_2^{'}) : (x_3, x_4^{'}, x_3, x_4^{'}) \right)$ $T_{b_{12}} = 5, 47$ bit/sample time. Since $T_{b_{12}} \ll T_{w_1} + T_{w_2} \Rightarrow$ $5, 47 \ll 13, 47 + 16, 63$, the calculations confirm that the complex system is indeed nearly decomposable into subsystems S^1 and S^2.

4.2. Decentralized control of complex system

After decomposition of complex system into subsystems, we are able to design decentralized controller for each subsystem. In this section, two basic methods of decentralized control are introduced, namely decentralized control with quadratic criterion (partially decentralized control) and totally decentralized control.

4.2.1. Decentralized control with quadratic criterion

If we have complex system (5), so we know that is decomposable into two subsystems and thus global criterion is $J = \sum_{i=1}^{2} J_i$, while the local criterion is

$$J_i = \frac{1}{2} \int_0^{\infty} \left(\left\langle x_i(t), Q_i x_i(t) \right\rangle + \left\langle u_i(t), R_i u_i(t) \right\rangle \right) dt \text{, for } i = 1, 2.$$

Control strategy for first subsystem is in the form $u_1(t) = -R_1^{-1} B_1^T K_1 \overline{x}_1(t) + R_1^{-1} B_1^T h_1$ and for second subsystem in the form $u_2(t) = -R_2^{-1} B_2^T K_2 \overline{x}_2(t) + R_2^{-1} B_2^T h_2$, where K_1 and K_2 are solution of Riccati equations (12)

$$\dot{K}_i = -K_i A_i - A_i^T K_i + K_i B_i R_i^{-1} B_i^T K_i - Q_i \qquad (12)$$

which are calculated off-line, h_1 and h_2 are solution of (13) and are calculated on-line.

$$\dot{h}_i = -(A_i - B_i R_i^{-1} B_i^T K_i)^T h_i + \\ + K_i \sum_{\substack{j=1 \\ j \neq i}}^{N} A_{ij} x_j(t) + \sum_{\substack{j=1 \\ j \neq i}}^{N} A_{ji}^T (K_j x_j(t) - h_j) \qquad (13)$$

Proposed decentralized control with quadratic criterion can then be applied to control complex system (5). The main task of this control strategy is to convert the system from given initial conditions to steady state with minimizing a given criterion function. It should be noted, that control is designed to compensate the effect of interactions that exist between subsystems, like compensation of measurable disturbances in optimal control systems. States of complex system controlled by decentralized control with quadratic criterion are shown on figure (see Fig. 4).
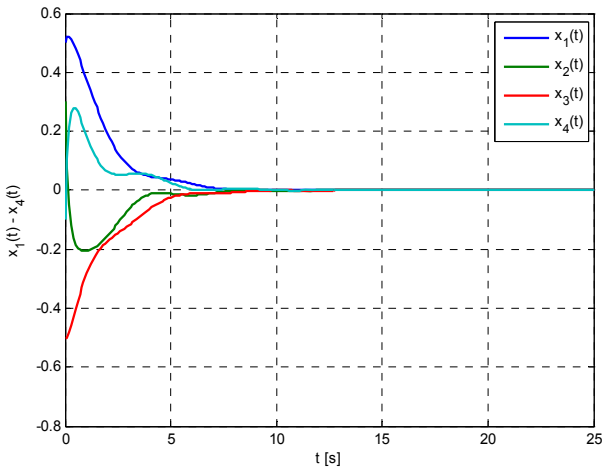
Fig. 4 States of complex system – decentralized control with quadratic criterion

4.2.2. Totally decentralized control

If we have complex system (5), so we know that is decomposable into two subsystems and thus global criterion is $J = \sum_{i=1}^{2} J_i$, while the local criterion is

$$J_i = \frac{1}{2} \int_0^\infty \left(\langle x_i(t), Q_i x_i(t) \rangle + \langle u_i(t), R_i u_i(t) \rangle \right) dt, \text{ for } i = 1, 2.$$

Control strategy for first subsystem is in the form $u_1(t) = -R_1^{-1} B_1^T K_1 \overline{x}_1(t)$ and for second subsystem in the form $u_2(t) = -R_2^{-1} B_2^T K_2 \overline{x}_2(t)$, where K_1 and K_2 are solution of Riccati equations (12).
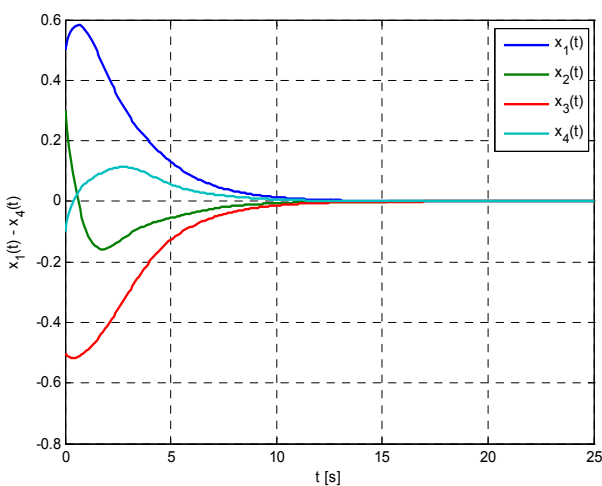

Fig. 5 States of complex system – totally decentralized control

States of complex system, which is controlled by totally decentralized control, are shown on figure (see Fig. 5). In this method of decentralized control, subsystems are considered as completely independence i.e. without interactions, while in a complex system are present. The disadvantage of this method is that it must be ensured not only the stability of subsystems, but also the stability of a complex system as a whole.

4.3. Decentralized control and information theory

As seen on figure (see Fig.4 and Fig. 5) from previous chapter, both methods of decentralized control convert complex system from specified initial conditions to steady state with minimizing a given criterion function. Let's look on these methods from information theory point of view. For better review and subsequent analysis will be better, that we look on internal structure of complex system (5), as shown on figure (see Fig. 6) [9].
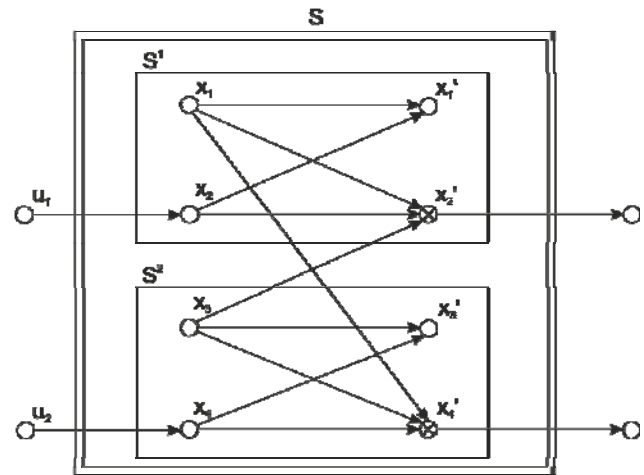

Fig. 6 Internal structure of complex system – information theory point of view

Figure (see Fig. 6) captures the internal structure of a complex system in discrete form (6) if the system is deterministic and its evolution can be represent by deterministic function $x(k+1) = f\left(x(k) \right)$.

In this case Deterministic Partition Law of Information Rates (DPLIR) proposed by Roger C. Conant [2] is an appropriate tool to analyze complex systems in terms of information and information structure. Our complex system (5) respectively (6) is dynamic system, which changing its properties in time, so we will try to adapt this law to its properties [9]. Of course we assume that the complex system is decomposed into subsystems as shown on the figure (see Fig. 6) [9].

Let's start with the first subsystem, labelled as S^1. According to the figure (see Fig. 6), system contains internal and output variables. Sets of output variables are denoted as S_0^i and sets of internal variables are denoted as S_{int}^i for i-th subsystem. So $S_0^1 = \{x_2'\}$, $S_{int}^1 = \{x_1, x_2, x_1'\}$ and $S^1 = \{x_1, x_2, x_1', x_2'\}$ for the first subsystem S^1. For total rate (of information flow) F^1 in the first subsystem we can write

$$F^1 = \sum_j H(x_j^1) = H(x_1) + H(x_2) + H(x_1') + H(x_2') \quad (14)$$

Throughput rate F_t^1 can be expressed as

$$F_t^1 = H(S_0^1) = H(x_2') \quad (15)$$

For blockage rate F_b^1 we obtain

$$
\begin{aligned}
F_b^1 &= H(S_{\text{int}}^1 \mid S_0^1) = H(S_{\text{int}}^1, S_0^1) - H(S_0^1) = \\
&= H(x_1, x_2, x_1', x_2') - H(x_2')
\end{aligned}
\tag{16}
$$

Finally, coordination rate F_c^1 (of information flow) in the first subsystem can be written as

$$
\begin{aligned}
F_c^1 &= H(x_1) + H(x_2) + H(x_1') + H(x_2') - \big[H(x_1) + \\
&\quad + H(x_2) + H(x_1') + H(x_2') - T(x_1 : x_2 : x_1' : x_2') \big] = \\
&= T(x_1 : x_2 : x_1' : x_2')
\end{aligned}
\tag{17}
$$

Second subsystem denoted as S^2 have output and internal variables, that forming a sets $S_0^2 = \{x_4'\}$ and $S_{\text{int}}^2 = \{x_3, x_4, x_3'\}$. For total rate (of information flow) F^2 in the first subsystem we can write

$$
F^2 = \sum_j H(x_j^2) = H(x_3) + H(x_4) + H(x_3') + H(x_4')
\tag{18}
$$

Throughput rate F_t^2 can be expressed as

$$
F_t^2 = H(S_0^2) = H(x_4')
\tag{19}
$$

For blockage rate F_b^2 we obtain

$$
\begin{aligned}
F_b^2 &= H(S_{\text{int}}^2 \mid S_0^2) = H(S_{\text{int}}^2, S_0^2) - H(S_0^2) \\
&= H(x_3, x_4, x_3', x_4') - H(x_4')
\end{aligned}
\tag{20}
$$

Finally, coordination rate F_c^2 (of information flow) in the first subsystem can be written as

$$
\begin{aligned}
F_c^2 &= H(x_3) + H(x_4) + H(x_3') + H(x_4') - \big[H(x_3) + \\
&\quad + H(x_4) + H(x_3') + H(x_4') - T(x_3 : x_4 : x_3' : x_4') \big] = \\
&= T(x_3 : x_4 : x_3' : x_4')
\end{aligned}
\tag{21}
$$

Table (see Table 7) shows and summarizes results of computation of Deterministic Partition Law of Information Rates for Dynamical Systems (DPLIRDS) derived in [9]. Units for all rates are bit per sample time.

Table 7 Decentralized control in the terms of Deterministic Partition Law of Information Rates

Decentralized control	F^1	F_t^1	F_b^1	F_c^1
Partially	7,9770	1,9063	0,6382	5,5325
Totally	11,681	2,7987	0,9549	7,9276
	F^2	F_t^2	F_b^2	F_c^2
Partially	7,6910	1,6625	0,8075	5,2210
Totally	12,435	3,0909	0,8195	8,5250

5. CONCLUSIONS

Based on above derived and obtained law we can draw some remarks and conclusions. Consider figure (see Fig. 6), if we have full knowledge of inputs, then each state can be seen as output of communication channel. Uncertainty of output of communication channel can be affected only by change of the statistical properties of its input, which in our case corresponds to two different methods of decentralized control. We considered that uncertainty might be represented by computation of entropy. Figures (see Fig. 4 and Fig. 5) shown that uncertainties of all states with partially decentralized control are smaller then with totally decentralized control. The sum of all these uncertainties corresponds with total rate (of information flow). Of course not every state of the system must be its output and that is why the throughput rate only measures uncertainty of outputs variables. Blockage rate in control systems is negligible, because control systems don't process information directly and blocking of irrelevant information in the internal structure of system do not play major role, as confirmed by our results (see Table 7). Coordination rate has the greatest value of all rates. The main reason is in the decomposition, because the main idea of decomposition as mentioned above, assumes that the relationship between variables within the subsystem must be much larger than relationship between whole subsystems. The obtained results suggest that in partially decentralized control coordination rate is much smaller than in totally decentralized control. The reason is very simple. In partially decentralized control exists information link between controllers, i.e. controller for first subsystem has information from controller of second subsystem and vice versa. On the other hand in totally decentralized control this link no exists, so decentralized controllers control subsystems as independent parts and that is why the coordination rate must increase. Similar conclusions can be done from comparison of numerical criteria of control performance quality. But these criteria cannot explain why these differences occur and method presented in this paper can explain this.

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BIOGRAPHIES

Ľuboš Popovič was born on 5.6.1984. In 2007 he graduated (MSc) at the Department of Cybernetics and Artificial Intelligence of the Faculty of Electrical Engineering and Informatics at Technical University in Košice. Since 2007 he is a PhD student at the Department of Cybernetics and Artificial Intelligence. His PhD thesis in the field of Cybernetics; is "Principle of requisite variety in control systems". His scientific research is focusing on dynamic and control of complex and hybrid systems.

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